STATISTICS · STUDY GUIDE

Exam 3 Study Guide

This guide covers chapters 5 and 6, the language of chance. You will list sample spaces, use the addition, multiplication and complement rules, work with conditional probability and read probabilities from two-way tables and trees. Then you will turn outcomes into numbers with random variables, check that a distribution is valid, find expected values, and use the binomial, Poisson and geometric models. Work one chapter at a time, try each worked example with the steps covered, and finish with the ten self-test items. The cram sheet at the end holds every formula.

WHAT THIS GUIDE COVERS

- Chapter 5 · sample spaces and events, classical probability, the complement, addition and multiplication rules, mutually exclusive versus independent events, conditional probability, two-way tables, trees and reversing a condition.
- Chapter 6 · discrete and continuous random variables, valid distributions, expected value and standard deviation, the binomial model with $\mu = np$ and $\sigma = \sqrt{(npq)}$, the Poisson and geometric models, and how to choose among them.

CHAPTER 5

Probability Basics

Read it in full: **Chapter 5 in the textbook**. Practice it in the lab: **Sample Space** and **Two-Way Tables**.

What to know

A **random process** has a known list of possible results but an unknown result this time. Each result is an **outcome**, and the complete, non-overlapping list is the **sample space** S. An **event** is any subset of S. Two coin flips give S = {HH, HT, TH, TT}: four outcomes, not three, because HT and TH are different. Two dice give $6 \times 6 = 36$ ordered pairs.

When outcomes are equally likely, P(A) = (outcomes in A) ÷ (outcomes in S). Every probability lies between 0 and 1, P(S) = 1, and the **complement rule** says $P(A^c) = 1 - P(A)$. With two dice, six pairs sum to 7, so P(7) = 6/36 = 1/6. A probability is a long-run relative frequency. The **law of large numbers** says observed proportions settle toward the true probability, but nothing is ever "due": a fair coin after ten tails is still 0.5.

"Or" is inclusive. The **general addition rule** is P(A or B) = P(A) + P(B) − P(A and B); the subtraction removes the overlap counted twice. **Mutually exclusive** (disjoint) events cannot happen together, so P(A and B) = 0 and P(A or B) = P(A) + P(B). For one card, P(heart or face card) = 13/52 + 12/52 − 3/52 = 22/52.

The **general multiplication rule** is P(A and B) = P(A) × P(B | A). Events are **independent** when P(B | A) = P(B), so then P(A and B) = P(A) × P(B). That equation is also the test: if P(A and B) differs from P(A)P(B), the events are dependent. Sampling without replacement creates dependence: two hearts in a row is (13/52)(12/51) ≈ 0.0588 without replacement and $(13/52)^2$ = 0.0625 with it. Below about 10% of a population, treating draws as independent is a fair approximation.

Mutually exclusive is not independent. If two events with nonzero probability are disjoint, knowing one happened tells you the other did not, so they are dependent. "At least one" problems are complement problems: P(at least one) = 1 − P(none). Four rolls of a die give at least one six with probability $1 - (5/6)^4 \approx 0.518$, not 4/6.

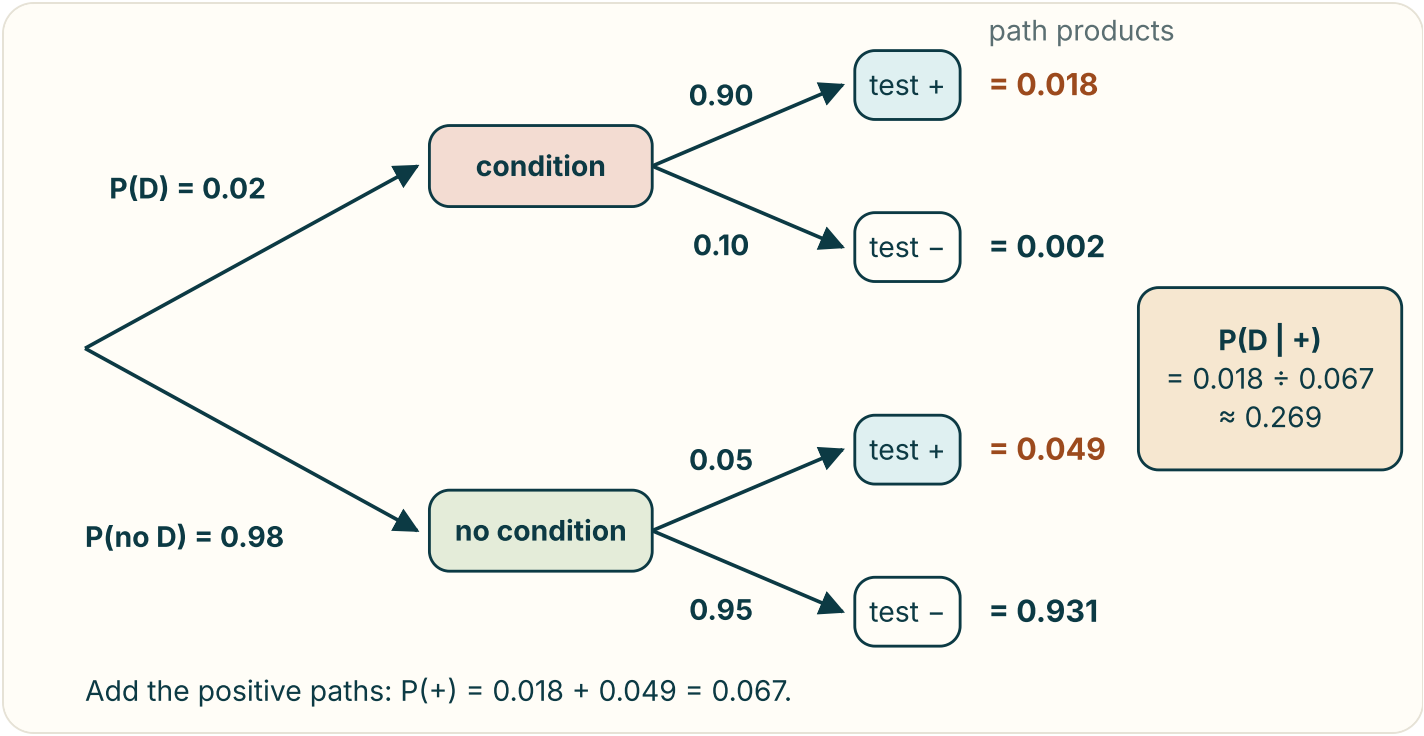
Conditional probability P(A | B) = P(A and B) ÷ P(B) shrinks the sample space to B. Told a die roll is even, the chance it is a 6 is 1/3. Reversing the bar changes the question: P(even | 6) = 1. Confusing P(A | B) with P(B | A) is the **confusion of the inverse**. A p-value is P(data this extreme | null true), not P(null true | data).

A **two-way table** gives every probability by choosing the denominator. **Joint** = cell ÷ grand total. **Marginal** = row or column total ÷ grand total. **Conditional** = cell ÷ its row or column total. In the book's 300 applicants, P(offer | certified) = 90/150 = 0.600 while P(certified | offer) = 90/135 ≈ 0.667. Since P(certified)P(offer) = 0.225 differs from the joint 0.300, the two are dependent, though the table cannot show that certification caused the offers.

A **tree diagram** stages the same facts. Multiply along a path to get its probability; add across paths that end the same way. Path probabilities must sum to 1. Adding paths gives the **law of total probability**, and dividing the path you want by the total of all paths with that ending reverses the conditioning. That reversal is the engine behind Bayes' rule.

LOCK THESE IN

- P(A) = outcomes in A ÷ outcomes in S, for equally likely outcomes only.
- Complement: $P(A^c) = 1 - P(A)$. "At least one" = 1 − P(none).
- Addition: P(A or B) = P(A) + P(B) − P(A and B).
- Multiplication: P(A and B) = P(A) × P(B | A); independent means P(B | A) = P(B).
- Mutually exclusive means P(A and B) = 0, and that makes the events dependent.
- Conditional: P(A | B) = P(A and B) ÷ P(B). Reversing the bar changes the answer.
- Joint ÷ grand total; marginal = margin ÷ grand total; conditional = cell ÷ its row or column.
- Trees: multiply along, add across, and the ends sum to 1.



Multiply along paths, add across paths. With 2% prevalence, a test that catches 90% of cases and gives 5% false positives yields P(+) = 0.067, and only about 27% of positives have the condition.

Formulas and when to use them

Rule	Formula	When to use it
Classical probability	$P(A) = n(A) \div n(S)$	Equally likely outcomes
Complement	$P(A^c) = 1 - P(A)$	"Not" and "at least one" questions
General addition	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	Any "or" question
Disjoint addition	$P(A \text{ or } B) = P(A) + P(B)$	Events that cannot co-occur
General multiplication	$P(A \text{ and } B) = P(A) \times P(B \mid A)$	Any "and" question, including without replacement
Independent multiplication	$P(A \text{ and } B) = P(A) \times P(B)$	Independent events; also the test for independence
Conditional	$P(A \mid B) = P(A \text{ and } B) \div P(B)$	"Given" questions
At least one in n independent tries	$1 - (1 - p)^n$	Repeated trials
Total probability	$P(B) = P(A)P(B \mid A) + P(A^c)P(B \mid A^c)$	Adding tree paths
Reverse a condition	$P(A \mid B) = \text{path}(A \text{ then } B) \div P(B)$	Screening, Bayes style questions

Words that decide the rule

Word or idea	Meaning	Check
Or	At least one happens (inclusive)	Subtract the overlap
And	Both happen	Multiply, with a condition if dependent
Mutually exclusive	Cannot happen together	P(A and B) = 0
Independent	One does not change the other	P(A | B) = P(A)
Given	Restrict to a smaller world	New denominator
Without replacement	The pool changes after each draw	Dependent draws
Law of large numbers	Long-run share approaches P	No "correction" in the short run

Worked examples

WORKED EXAMPLE 1 · COUNTING WITH TWO DICE

Roll two fair dice. Find P(sum = 8), P(sum ≤ 4) and P(sum is not 8).

STEP 1 · THE SAMPLE SPACE

Keep the dice distinct: 6 × 6 = 36 equally likely ordered pairs.

STEP 2 · SUM OF 8

(2,6), (3,5), (4,4), (5,3), (6,2): 5 pairs, so P = 5/36 ≈ **0.139**.

STEP 3 · SUM AT MOST 4

Sum 2: 1 pair; sum 3: 2 pairs; sum 4: 3 pairs; total 6. P = 6/36 = 1/6 ≈ **0.167**.

STEP 4 · COMPLEMENT

P(not 8) = 1 − 5/36 = 31/36 ≈ **0.861**.

WORKED EXAMPLE 2 · THE GENERAL ADDITION RULE WITH CARDS

Draw one card from a standard 52-card deck. Find P(red or king).

STEP 1 · RED

Hearts and diamonds: 26 cards, so P(red) = 26/52.

STEP 2 · KING

Four kings, so P(king) = 4/52.

STEP 3 · OVERLAP

The red kings are the king of hearts and king of diamonds: 2/52.

STEP 4 · COMBINE

26/52 + 4/52 − 2/52 = 28/52 = 7/13 ≈ **0.538**.

CHECK BY COUNTING

26 red cards plus the 2 black kings is 28 cards. Forgetting the subtraction gives 30/52, which counts the two red kings twice.

WORKED EXAMPLE 3 · WITH AND WITHOUT REPLACEMENT, AND AT LEAST ONE

Draw two cards. Find P(both aces) with and without replacement. Then find the chance of at least one six in three rolls of a die.

STEP 1 · WITHOUT REPLACEMENT

(4/52)(3/51) = 12/2,652 = 1/221 ≈ **0.0045**. The second draw faces 3 aces in 51 cards.

STEP 2 · WITH REPLACEMENT

(4/52)(4/52) = 1/169 ≈ **0.0059**. The draws are independent.

STEP 3 · AT LEAST ONE SIX

P(no six in 3) = $(5/6)^3$ = 125/216 ≈ 0.5787. So P(at least one) = 1 − 125/216 = 91/216 ≈ **0.421**.

WHY NOT 3/6?

Adding 1/6 three times assumes the events cannot overlap. Two sixes in three rolls would be counted twice, and six rolls would give an impossible 6/6 for "certain."

WORKED EXAMPLE 4 · A TWO-WAY TABLE, START TO FINISH

400 adults are classified by flu shot and whether they caught the flu: vaccinated with flu 24, vaccinated without 216, unvaccinated with flu 36, unvaccinated without 124.

STEP 1 · MARGINS

Vaccinated 240, unvaccinated 160; caught flu 60, did not 340; total 400.

STEP 2 · JOINT AND MARGINAL

P(vaccinated and flu) = 24/400 = 0.060. P(vaccinated) = 240/400 = 0.600. P(flu) = 60/400 = 0.150.

STEP 3 · CONDITIONAL

P(flu | vaccinated) = 24/240 = **0.100**. P(flu | unvaccinated) = 36/160 = **0.225**.

STEP 4 · INDEPENDENCE TEST

P(vaccinated) × P(flu) = 0.600 × 0.150 = 0.090, but the joint is 0.060. They differ, so the variables are dependent.

STEP 5 · REVERSE THE BAR

P(vaccinated | flu) = 24/60 = 0.400. Same cell, different denominator, different question. The table shows association; only an experiment could show the shot caused the difference.

TRAPS PEOPLE FALL INTO

- **Listing three outcomes for two coins.** HT and TH are different outcomes. S has four.
- **Thinking an outcome is "due."** Independent trials have no memory. The law of large numbers works by dilution, not correction.
- **Forgetting to subtract the overlap.** P(A or B) needs − P(A and B) unless the events are disjoint.
- **Calling disjoint events independent.** Disjoint events with nonzero probability are dependent.
- **Multiplying when events are dependent.** Use P(B | A), especially when sampling without replacement.
- **Adding probabilities across repeated tries.** Use 1 − P(none) for "at least one."
- **Swapping P(A | B) and P(B | A).** Ask "what is my denominator now?" before you divide.
- **Reading cause from a two-way table.** A table shows association. Cause needs random assignment.

Chapter 5 self-test · ten items

1. Three fair coins are flipped. How many outcomes are in the sample space?

A. 4

B. 6

C. 3

D. 8

D. 8 Each coin has 2 outcomes, so 2 × 2 × 2 = 8: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

2. Two dice are rolled. What is P(sum = 10)?

A. 1/36
B. 2/36
C. 3/36
D. 4/36

C. 3/36 (4,6), (5,5) and (6,4) give 10, so 3 of 36 equally likely pairs.

3. P(A) = 0.5, P(B) = 0.4 and P(A and B) = 0.2. What is P(A or B)?

A. 0.7
B. 0.9
C. 0.2
D. 1.1

A. 0.7 0.5 + 0.4 − 0.2 = 0.7. Without the subtraction you get 0.9.

4. A and B are mutually exclusive, each with probability above 0. Which is true?

A. They are independent
B. P(A or B) = P(A)P(B)
C. P(A and B) = P(A)P(B)
D. They are dependent

D. They are dependent If A happens, B cannot, so knowing A changes P(B) to 0. That is dependence.

5. Two cards are drawn without replacement. What is P(both are spades)?

A. (13/52)(12/51) ≈ 0.0588
B. (13/52)(13/52) = 0.0625
C. (13/52) + (12/51) ≈ 0.485
D. (13/52)(12/52) ≈ 0.0577

A. (13/52)(12/51) ≈ 0.0588 After one spade is removed, 12 spades remain among 51 cards, so use P(A) × P(B | A).

6. A free-throw shooter makes 70% of shots, independently. What is P(at least one miss in 3 shots)?

A. 0.900
B. 0.027
C. 0.657
D. 0.343

C. 0.657 P(no misses) = 0.7^3 = 0.343, so P(at least one miss) = 1 − 0.343 = 0.657.

7. A die shows a number greater than 3. What is P(it is a 6)?

A. 1/6
B. 1/3
C. 1/2
D. 1/4

B. 1/3 The given event is {4, 5, 6}. One of those three is a 6.

8. In a table, 60 of 200 people exercise, 45 of whom sleep well. What is P(sleeps well | exercises)?

A. 0.225
B. 0.75
C. 0.30
D. 0.45

B. 0.75 Restrict to the 60 who exercise: 45 ÷ 60 = 0.75. Dividing 45 by 200 gives the joint probability, 0.225.

9. P(A) = 0.3, P(B) = 0.5 and P(A and B) = 0.15. Are A and B independent?

A. Yes, because 0.3 × 0.5 = 0.15
B. No, because 0.3 + 0.5 is not 1
C. No, because P(A and B) is above 0
D. Yes, because P(A) is less than P(B)

A. Yes, because 0.3 × 0.5 = 0.15 Independence holds exactly when P(A and B) = P(A)P(B), and 0.3 × 0.5 = 0.15.

10. In a tree, P(A) = 0.4, P(B | A) = 0.5 and P(B | not A) = 0.25. What is P(B)?

A. 0.75
B. 0.20
C. 0.15
D. 0.35

D. 0.35 Add the two paths that end in B: (0.4)(0.5) + (0.6)(0.25) = 0.20 + 0.15 = 0.35.

Discrete Random Variables

Read it in full: **Chapter 6 in the textbook**. Practice it in the lab: **Expected Value** and **Binomial Distribution**.

What to know

A **random variable** attaches a number to each outcome of a chance process. Capital X names the variable; lowercase x names one of its values, so $P(X = x)$ is the chance X equals that number. A random variable is **discrete** if its values can be listed (counts), even if the list never ends, and **continuous** if it fills an interval (measurements). For a continuous variable the probability of any single exact value is 0, so we ask about intervals.

A **probability distribution** lists every value with its probability. It is valid only if $\Sigma P(x) = 1$ (allowing for rounding) and $0 \le P(x) \le 1$ for every x. A table that sums to 0.90 has lost an outcome; a table with a negative entry is invalid even if it sums to 1. One missing probability can always be recovered as 1 minus the rest. Compound events are sums, because the values are mutually exclusive.

The mean or **expected value** is $\mu = E(X) = \Sigma[x \cdot P(x)]$, a probability-weighted average with no final division. It describes the long run, not the next trial, and it need not be a possible value. The variance is $\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$, or the shortcut $\sigma^2 = \Sigma[x^2 \cdot P(x)] - \mu^2$. The square root σ returns to the original units. By the **range rule of thumb**, values outside $\mu \pm 2\sigma$ are unusual.

Expected value prices bets and policies. In the book's raffle, 500 tickets at \$3 with one \$1,000 prize give a net gain of \$997 with probability 1/500 and −\$3 with probability 499/500, so $E(X) =$ −\$1.00 per ticket. That is what the buyer loses on average and what the club clears on average.

A **binomial** experiment needs all four: a fixed number of trials n, two outcomes per trial, independent trials and a constant success probability p. "Success" is just the outcome you count. Then $P(X = x) = {}_nC_x\, p^x\, q^{n-x}$, with $q = 1 - p$ and ${}_nC_x = n! \div [x!(n - x)!]$. The coefficient counts arrangements; $p^x q^{n-x}$ is the chance of one arrangement. A student guessing on 6 four-choice questions gets exactly 2 right with probability 0.2966.

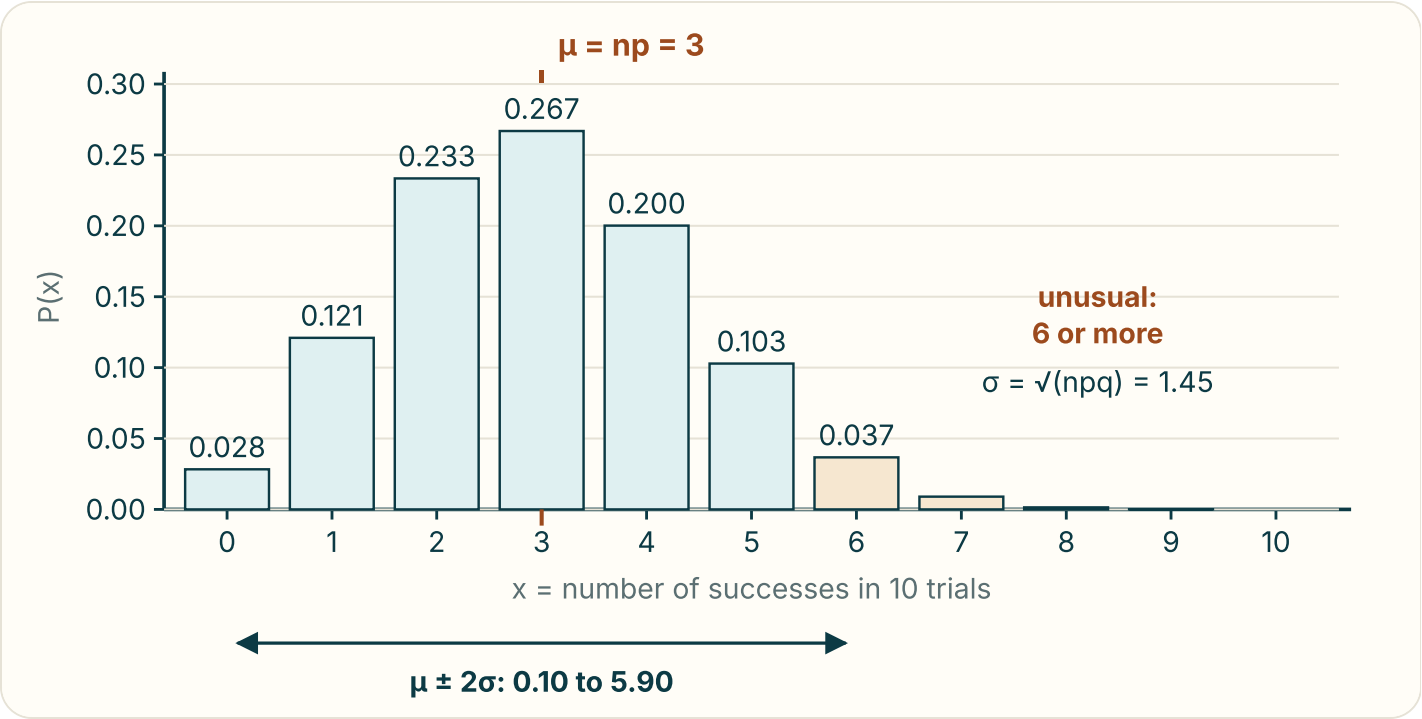
For a binomial, $\mu = np$ and $\sigma = \sqrt{(npq)}$, exactly. The guessing student averages 1.5 correct with $\sigma \approx 1.06$, so 4 or more correct is unusual, and the exact tail $P(X \geq 4) = 0.0376$ agrees. Sampling without replacement is treated as binomial when the sample is no more than 5% of the population.

The **Poisson** distribution counts events in a fixed interval at a constant average rate λ: $P(X = x) = \lambda^x e^{-\lambda} \div x!$, for x = 0, 1, 2, and so on. Its mean and variance are both λ, so $\sigma = \sqrt{\lambda}$. With $\lambda = 3$ calls per hour, P(5 calls) ≈ 0.1008. It approximates a binomial with $\lambda = np$ when n is large and p small ($n \geq 100$ and $np \leq 10$).

The **geometric** distribution counts trials until the first success: $P(X = x) = q^{x-1}p$ for x = 1, 2, 3 and so on, with $\mu = 1 \div p$ and $\sigma = \sqrt{q} \div p$. With p = 0.20 the first success lands on call 4 with probability 0.1024, and it takes 5 calls on average. $P(X > k) = q^k$. The geometric is memoryless: past failures build no credit. To choose a model, ask what is fixed: the trials (binomial), the interval (Poisson) or the single success (geometric).

LOCK THESE IN

- Valid distribution: $\Sigma P(x) = 1$ and $0 \leq P(x) \leq 1$ for every x.
- $\mu = E(X) = \Sigma x \cdot P(x)$; $\sigma^2 = \Sigma x^2 \cdot P(x) - \mu^2$; $\sigma = \sqrt{\sigma^2}$.
- Expected value is the long-run average, not the next result.
- Binomial needs fixed n, two outcomes, independence and constant p.
- $P(X = x) = {}_nC_x\, p^x\, q^{n-x}$; $\mu = np$; $\sigma = \sqrt{(npq)}$.
- Poisson: $P(x) = \lambda^x e^{-\lambda} \div x!$; mean = variance = λ.
- Geometric: $P(x) = q^{x-1}p$; $\mu = 1/p$; $P(X > k) = q^k$.
- Unusual: outside $\mu \pm 2\sigma$.



A binomial distribution drawn to scale. With n = 10 and p = 0.3 the bars peak at 3, the mean np. The standard deviation is √(10 × 0.3 × 0.7) ≈ 1.45, so counts of 6 or more fall outside μ ± 2σ.

Formulas and when to use them

Quantity	Formula	When to use it
Validity	$\Sigma P(x) = 1$; $0 \le P(x) \le 1$	Checking any discrete table
Mean	$\mu = \Sigma[x \cdot P(x)]$	Long-run average of a random variable
Variance	$\sigma^2 = \Sigma[x^2 \cdot P(x)] - \mu^2$	Spread; take √ for σ
Unusual values	Outside $\mu \pm 2\sigma$	Range rule of thumb
Binomial probability	${}_nC_x\, p^x\, q^{n-x}$	Fixed n, two outcomes, independent, constant p
Combinations	${}_nC_x = n! \div [x!(n - x)!]$	Counting arrangements
Binomial mean and SD	$\mu = np$; $\sigma = \sqrt{(npq)}$	Exact shortcuts
Poisson probability	$\lambda^x e^{-\lambda} \div x!$	Events in a fixed interval at rate λ
Poisson mean and SD	$\mu = \sigma^2 = \lambda$; $\sigma = \sqrt{\lambda}$	Variance far above the mean means Poisson fits poorly
Geometric probability	$q^{x-1}\, p$	Trial of the first success
Geometric mean and SD	$\mu = 1/p$; $\sigma = \sqrt{q} \div p$	Average wait for the first success

Which model?

Model	What is fixed	What is random
Binomial	Number of trials n	Number of successes, 0 to n
Poisson	The interval (time, area, length)	Number of events, 0, 1, 2, ...
Geometric	One success	Number of trials needed, 1, 2, 3, ...

Worked examples

WORKED EXAMPLE 1 · A MISSING PROBABILITY, THE MEAN AND THE SD

X is the number of streaming services a household pays for, with P(0) = 0.10, P(1) = 0.25, P(2) = k, P(3) = 0.20, P(4) = 0.10. Find k, μ and σ.

STEP 1 · FIND K

0.10 + 0.25 + k + 0.20 + 0.10 = 1, so 0.65 + k = 1 and **k = 0.35**, a legal probability.

STEP 2 · MEAN

(0)(0.10) + (1)(0.25) + (2)(0.35) + (3)(0.20) + (4)(0.10) = **μ = 1.95**.

STEP 3 · ΣX²P(X)

(0)(0.10) + (1)(0.25) + (4)(0.35) + (9)(0.20) + (16)(0.10) = 5.05.

STEP 4 · VARIANCE AND SD

$\sigma^2 = 5.05 - 1.95^2 = 5.05 - 3.8025 = 1.2475$, so $\sigma \approx$ **1.12**.

STEP 5 · UNUSUAL?

μ ± 2σ = −0.28 to 4.18. Every possible value lies inside, so none is unusual.

WORKED EXAMPLE 2 · IS THE EXTENDED WARRANTY WORTH IT?

A $60 warranty covers a repair that costs $450 and is needed with probability 0.08 in the warranty period. Find the expected cost of buying versus not buying.

STEP 1 · BUY IT

You pay $60 for sure. Expected cost = **$60**.

STEP 2 · SKIP IT

Cost is $450 with probability 0.08 and $0 otherwise: E = 450(0.08) + 0(0.92) = **$36**.

STEP 3 · COMPARE

Skipping saves $24 per item on average. From the seller's side, E(profit) = 60 − 36 = $24 per warranty.

READ IT RIGHT

Expected value is the long-run average over many purchases. A person who cannot absorb a large surprise bill might still choose the warranty; the number prices the bet, it does not make the choice.

WORKED EXAMPLE 3 · A BINOMIAL FROM START TO FINISH

A clinic finds that 20% of patients miss appointments, independently. Of the next 8 patients, find P(exactly 2 miss), P(at least 1 misses), μ and σ.

STEP 1 · CHECK THE FOUR

n = 8 fixed; miss or not; independent; p = 0.2 each time. Binomial.

STEP 2 · EXACTLY 2

$_8C_2 = 28$. $28 \times 0.2^2 \times 0.8^6 = 28 \times 0.04 \times 0.262144 =$ **0.2936**.

STEP 3 · AT LEAST 1

$P(0) = 0.8^8 = 0.1678$, so $P(X \geq 1) = 1 - 0.1678 =$ **0.8322**.

STEP 4 · MEAN AND SD

$\mu = np = 8(0.2) =$ **1.6**. $\sigma = \sqrt{(8 \times 0.2 \times 0.8)} = \sqrt{1.28} \approx$ **1.13**.

STEP 5 · UNUSUAL

$\mu + 2\sigma = 3.86$, so 5 or more no-shows out of 8 would be unusual.

WORKED EXAMPLE 4 · POISSON AND GEOMETRIC

A web page gets $\lambda = 2.5$ error reports per day. Find P(exactly 4) and P(none). Separately, 15% of cold emails get a reply; find P(first reply is the 3rd email) and the mean wait.

STEP 1 · POISSON, X = 4

$2.5^4 = 39.0625$; $e^{-2.5} = 0.082085$; $4! = 24$. $P = 39.0625 \times 0.082085 \div 24 =$ **0.1336**.

STEP 2 · POISSON, X = 0

$\lambda^0 = 1$ and $0! = 1$, so $P(0) = e^{-2.5} =$ **0.0821**; $\sigma = \sqrt{2.5} \approx 1.58$.

STEP 3 · GEOMETRIC, X = 3

Two misses then a reply: $0.85^2 \times 0.15 = 0.7225 \times 0.15 =$ **0.1084**.

STEP 4 · AVERAGE WAIT

$\mu = 1/p = 1/0.15 \approx$ **6.67 emails**, counting the email that gets the reply. P(more than 3 needed) $= 0.85^3 = 0.6141$.

TRAPS PEOPLE FALL INTO

- **Thinking discrete means "few values."** An endless list 0, 1, 2, ... is still discrete.
- **Checking only the sum.** A table that sums to 1 with a negative entry is invalid. Check both rules.
- **Dividing by n after Σx·P(x).** The weights already sum to 1. No final division.
- **Reading E(X) as the next result.** It is the long-run average and may be impossible on one trial.
- **Using the binomial without fixed n.** "Until the first success" is geometric, and "per hour" with no cap is Poisson.
- **Forgetting the coefficient.** $p^x q^{n-x}$ is one arrangement. Multiply by $_nC_x$.
- **Mismatching p and "success."** If you count defects, p is the chance of a defect.

- **Believing a success is overdue.** The geometric is memoryless; past failures change nothing.

Chapter 6 self-test · ten items

1. Which random variable is continuous?

A. The time a battery lasts
B. The number of calls in an hour
C. The number of heads in 5 flips
D. The number of defects in a lot

A. The time a battery lasts Time is measured and fills an interval. The others are counts.

2. A table gives P(0) = 0.2, P(1) = 0.5, P(2) = 0.4, P(3) = −0.1. Is it valid?

A. Yes, because the entries sum to 1
B. No, because the sum is above 1
C. No, because one probability is negative
D. Yes, because there are four values

C. No, because one probability is negative The entries do sum to 1.0, but a probability can never be negative. Both rules must hold.

3. X takes 1, 2, 3 with probabilities 0.5, 0.3, 0.2. What is E(X)?

A. 2.0
B. 0.57
C. 1.7
D. 1.5

C. 1.7 (1)(0.5) + (2)(0.3) + (3)(0.2) = 0.5 + 0.6 + 0.6 = 1.7.

4. For the same X (values 1, 2, 3; probabilities 0.5, 0.3, 0.2), what is σ^2?

A. 0.61
B. 3.50
C. 0.78
D. 2.89

A. 0.61 $\Sigma x^2 P(x) = 0.5 + 1.2 + 1.8 = 3.5$, and $3.5 - 1.7^2 = 3.5 - 2.89 = 0.61$.

5. A $2 game pays $30 with probability 0.05 and nothing otherwise. What is the expected net gain per play?

A. $1.50
B. −$2.00
C. $28.00
D. −$0.50

D. −$0.50 Net gain is $28 with probability 0.05 and −$2 with probability 0.95: $28(0.05) - 2(0.95) = 1.40 - 1.90 = -0.50$.

6. Which setting is binomial?

A. Flipping until the first head appears
B. Counting typos on one printed page
C. Drawing 5 cards from a 10-card deck
D. Counting heads in 12 flips of a fair coin

D. Counting heads in 12 flips of a fair coin Twelve fixed, independent flips with constant p is binomial. "Until" is geometric, typos per page are Poisson, and 5 of 10 cards breaks independence.

7. n = 5 and p = 0.4. What is P(X = 2)?

A. 0.0346
B. 0.3456
C. 0.2304
D. 0.4000

B. 0.3456 ${}_5C_2 = 10$, and $10 \times 0.4^2 \times 0.6^3 = 10 \times 0.16 \times 0.216 = 0.3456$. Leaving out the 10 gives 0.0346.

8. A binomial has n = 50 and p = 0.2. What are μ and σ?

A. μ = 10, σ = 8.00
B. μ = 10, σ ≈ 2.83
C. μ = 2.5, σ ≈ 1.58
D. μ = 40, σ ≈ 2.83

B. μ = 10, σ ≈ 2.83 $\mu = 50 \times 0.2 = 10$; $\sigma = \sqrt{(50 \times 0.2 \times 0.8)} = \sqrt{8} \approx 2.83$.

9. Calls arrive at λ = 2 per hour. What is P(no calls in an hour)?

A. 0.5000
B. 0.2707
C. 0.1353
D. 0.0000

C. 0.1353 $P(0) = 2^0 e^{-2} \div 0! = e^{-2} \approx 0.1353$.

10. A shooter makes 25% of long shots, independently. What is the mean number of attempts to the first make?

A. 3

B. 4

C. 0.25

D. 1.33

B. 4 Geometric mean $\mu = 1/p = 1/0.25 = 4$ attempts, counting the make.

ONE PAGE

Cram sheet

Everything from chapters 5 and 6 in one place. Cover the right column and see if you can rebuild each row.

Formulas

Name	Formula	Note
Classical	$P(A) = n(A) \div n(S)$	Equally likely outcomes
Complement	$P(A^c) = 1 - P(A)$	At least one = 1 − P(none)
Addition	$P(A) + P(B) - P(A \text{ and } B)$	Drop the last term if disjoint
Multiplication	$P(A) \times P(B \mid A)$	$P(A)P(B)$ if independent
Conditional	$P(A \text{ and } B) \div P(B)$	New denominator
Expected value	$\Sigma x \cdot P(x)$	Long-run average
Variance	$\Sigma x^2 P(x) - \mu^2$	$\sigma = \sqrt{\sigma^2}$
Binomial	${}_nC_x p^x q^{n-x}$; $\mu = np$; $\sigma = \sqrt{(npq)}$	Four conditions
Poisson	$\lambda^x e^{-\lambda} \div x!$; $\mu = \sigma^2 = \lambda$	Fixed interval
Geometric	$q^{x-1}p$; $\mu = 1/p$	First success

Rules that are not formulas

Topic	The rule
Sample space	List ordered outcomes; HT and TH differ.
Long run	Proportions settle toward P; nothing is due.
Disjoint vs independent	Disjoint events with nonzero P are dependent.
Two-way table	Joint over total; marginal over total; conditional over its row or column.
Trees	Multiply along, add across; ends sum to 1.
Inverse	P(A | B) is not P(B | A).
Valid table	Sum to 1 and every entry in [0, 1].
Unusual	Outside $\mu \pm 2\sigma$.
Model choice	Fixed trials: binomial. Fixed interval: Poisson. One success: geometric.

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