COLLEGE ALGEBRA · STUDY GUIDE

Final Exam Study Guide

The final covers all fourteen chapters. Each chapter here is shorter than in the exam guides: three paragraphs of what to know, the points to lock in, one table, two worked examples, the traps and six new self-test items. If a chapter feels shaky, go back to its exam guide for the full treatment. The long cram sheet at the end pulls every formula in the course into one place. Work two chapters a day and the whole course fits into a week.

WHAT THIS GUIDE COVERS

- Foundations: chapters 1 and 2 (numbers, exponents, linear equations and inequalities).
- Lines and functions: chapters 3 and 4 (graphs, slope, the three forms, functions and transformations).
- Systems and algebraic expressions: chapters 5, 6, 7 and 8 (systems, polynomials and factoring, rational expressions, radicals).
- Functions in depth: chapters 9 and 10 (quadratics, polynomial and rational functions).
- Exponentials and logarithms: chapters 11, 12 and 13 (functions, properties, equations and models).
- Patterns: chapter 14 (sequences, series and the binomial theorem).
- Every chapter carries what to know, a diagram, a table, two worked examples, the traps and a six-item self-test, and the cram sheet holds all the formulas.

CHAPTER 1

Real Numbers and Exponents

Read it in full: **Chapter 1 in the textbook**. Practice it in the lab: **One Operation at a Time** and **Move the Point**.

What to know

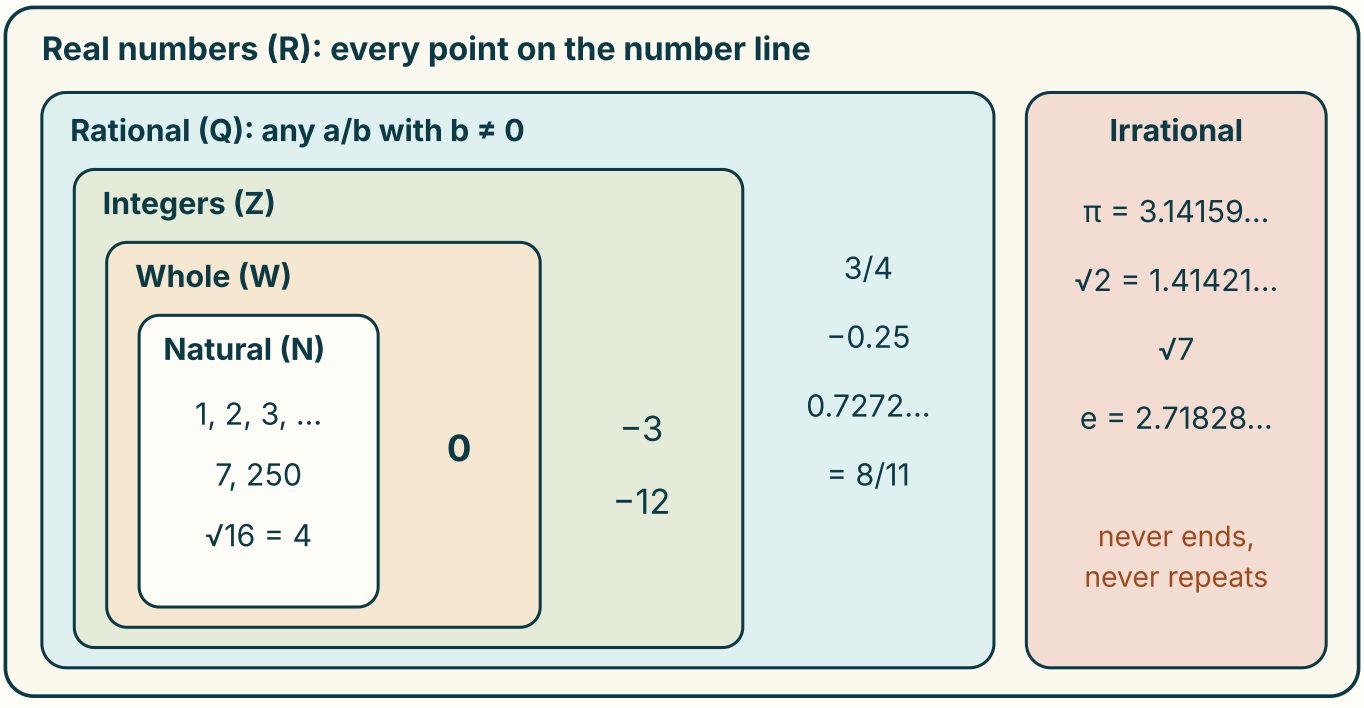
The number sets nest: natural ⊂ whole ⊂ integer ⊂ rational ⊂ real. A rational number is a/b with integers a and b and b ≠ 0; its decimal stops or repeats. An irrational number never stops and never repeats, such as π, e and √2, but √9 = 3 is rational. An even root of a negative is not real.

Commutative (order) and associative (grouping) work only for addition and multiplication. The distributive property $a(b + c) = ab + ac$ expands and factors. Division by zero is undefined. The order of operations has four levels: grouping, exponents, multiplication and division left to right, addition and subtraction left to right. $-3^2 = -9$ but $(-3)^2 = 9$.

Exponents add when you multiply like bases, subtract when you divide, and multiply for a power of a power. $a^0 = 1$ and $a^{-n} = 1/a^n$, both with $a \neq 0$. Powers never spread over sums. Scientific notation is $a \times 10^n$ with $1 \leq |a| < 10$: multiply the decimals and add exponents, then fix the form.

LOCK THESE IN

- $N \subset W \subset Z \subset Q \subset R$.
- Rational decimals stop or repeat.
- Four levels of operations; left to right within a level.
- Parenthesize every substituted negative.
- $a^0 = 1$; $a^{-n} = 1/a^n$.
- $1 \leq |a| < 10$ in scientific notation.



Each set holds every set drawn inside it. Not real: $\sqrt{(-4)}$.

Natural sits inside whole, inside integer, inside rational. Rationals and irrationals together fill the real line. A radical is irrational only when it does not come out evenly, so $\sqrt{16} = 4$ is a natural number.

Chapter 1 rules

Item	Formula or rule	When to use it
Product and quotient	$a^m a^n = a^{m+n}$; $a^m / a^n = a^{m-n}$	Like bases
Power rules	$(a^m)^n = a^{mn}$; $(ab)^n = a^n b^n$	Exponent outside parentheses
Zero and negative	$a^0 = 1$; $a^{-n} = 1/a^n$	$a \neq 0$
Scientific notation	$a \times 10^n$, $1 \leq \lvert a \rvert < 10$	Very large or small numbers

Worked examples

EXAMPLE 1.1 · ORDER OF OPERATIONS

Evaluate $30 \div 5 \times 2 - (-4)^2$.

STEP 1 · EXPONENT

$(-4)^2 = 16$.

STEP 2 · LEFT TO RIGHT

$30 \div 5 = 6$, then $6 \times 2 = 12$.

STEP 3 · SUBTRACT

$12 - 16 = \mathbf{-4}$.

EXAMPLE 1.2 · SCIENTIFIC NOTATION

Write $(2.5 \times 10^{-3})(6.0 \times 10^7)$ in proper form.

STEP 1 · MULTIPLY DECIMALS

$2.5 \times 6.0 = 15$.

STEP 2 · ADD EXPONENTS

$-3 + 7 = 4$, giving 15×10^4.

STEP 3 · FIX THE FORM

$\mathbf{1.5 \times 10^5}$.

TRAPS PEOPLE FALL INTO

- **Calling √16 irrational.** It equals 4.
- **Multiplying before dividing.** 8 ÷ 2 × 4 = 16.
- **Writing $-3^2 = 9$.** It is −9.
- **Spreading a power over a sum.** $(x + y)^2 \neq x^2 + y^2$.

Chapter 1 self-test · six items

1. Which of these has a decimal that never ends and never repeats?

A. √49

B. √20

C. −3/7

D. 0.125

B. √20 20 is not a perfect square, so √20 never ends or repeats. √49 = 7.

2. What is $-6^2 - (-6)^2$?

A. −72

B. 72

C. −36

D. 0

A. −72 $-6^2 = -36$ and $(-6)^2 = 36$, so −36 − 36 = −72.

3. Evaluate $10 - 2(3 - 5)^3 \div 4$.

A. −6

B. 6

C. 14

D. 2

C. 14 $(-2)^3 = -8$; 2(−8) = −16; −16 ÷ 4 = −4; 10 − (−4) = 14.

4. Simplify $(2x^3y^{-2})^{-2}$.

A. $-4x^6y^4$

B. $x^6/(4y^4)$

C. $4y^4/x^6$

D. $y^4/(4x^6)$

D. $y^4/(4x^6)$ $2^{-2}x^{-6}y^4 = y^4/(4x^6)$.

5. Express 0.0000703 as a $\times 10^n$ in proper form.

A. 7.03×10^{-5}

B. 7.03×10^5

C. 70.3×10^{-6}

D. 7.03×10^{-4}

A. 7.03×10^{-5} The decimal moves 5 places right, so the exponent is -5.

6. Which statement shows the additive inverse property?

A. $9 + 4 = 4 + 9$

B. $9 + (-9) = 0$

C. $9 \cdot 1 = 9$

D. $9 + 0 = 9$

B. $9 + (-9) = 0$ A number plus its opposite is 0.

CHAPTER 2

Linear Equations and Inequalities

Read it in full: **Chapter 2 in the textbook**. Practice it in the lab: **One, None, or Infinitely Many** and **Solve for a Variable**.

What to know

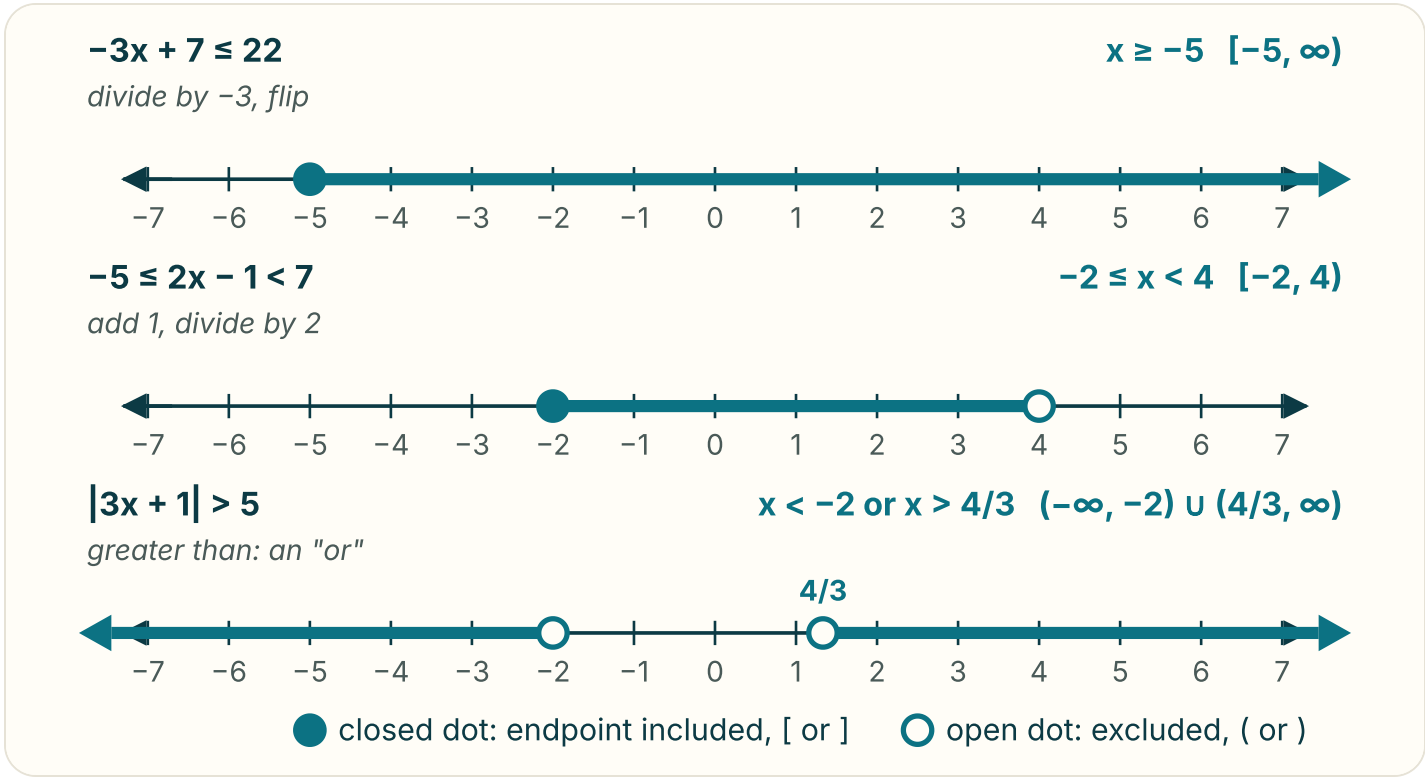
Solve a linear equation by doing the same legal move to both sides: distribute, combine, collect the variable, collect the numbers, divide, then check. Clear fractions with the LCD. If the variable is in a denominator, list restrictions first and discard extraneous answers.

A conditional equation has one solution, an identity has all reals (a true last line), and a contradiction has none, ∅ (a false last line). Solve formulas by undoing in reverse order; if the target letter appears twice, gather, factor and divide. In word problems, define the variable with units and answer the question asked.

Inequalities use the same moves, but multiplying or dividing by a negative flips the symbol. Brackets include endpoints, parentheses exclude them, and ∞ always takes a parenthesis. An "and" is an overlap; an "or" is a union. $|u| < c$ becomes $-c < u < c$, and $|u| > c$ becomes $u < -c$ or $u > c$, after isolating the bars.

LOCK THESE IN

- Same move on both sides; check in the original.
- Clear fractions with the LCD.
- True last line: all reals. False: ∅.
- Undo in reverse order for formulas.
- Flip only for × or ÷ by a negative.
- $|u| < c$: and. $|u| > c$: or.



Solutions are stretches of the line. A closed dot matches a bracket and an open dot matches a parenthesis. "Less than" in absolute value gives one middle piece; "greater than" gives two outward rays.

Chapter 2 rules

Item	Formula or rule	When to use it
Variable twice	$x = (d - b)/(a - c)$	$ax + b = cx + d$
Percent off	Sale = (1 − rate)p	Original price problems
Distance	$d = rt$	Opposite directions add
Absolute value	$\lvert u \rvert < c \Rightarrow -c < u < c$	Isolate the bars first

Worked examples

EXAMPLE 2.1 · A LINEAR EQUATION

Solve $2(x - 3) + 4 = 5x + 7$.

STEP 1 · DISTRIBUTE AND COMBINE

$2x - 2 = 5x + 7$.

STEP 2 · COLLECT

$-9 = 3x$, so **$x = -3$**.

CHECK

$2(-6) + 4 = -8$ and $5(-3) + 7 = -8$.

EXAMPLE 2.2 · A COMPOUND INEQUALITY

Solve $-4 < 3x + 2 \le 11$.

STEP 1 · SUBTRACT 2 FROM ALL THREE PARTS

$-6 < 3x \le 9$.

STEP 2 · DIVIDE BY 3

$-2 < x \le 3$, which is **$(-2, 3]$**.

TRAPS PEOPLE FALL INTO

- **Writing x = 0 for no solution.** A false last line means ∅.
- **Keeping an excluded value.** It is extraneous.
- **Flipping after subtracting.** Only × or ÷ by a negative flips.
- **Splitting before isolating the bars.** Isolate |u| first.

Chapter 2 self-test · six items

1. Solve $x/3 - 2 = x/4$.

A. $x = 24$

B. $x = 2$

C. $x = 12$

D. $x = -24$

A. $x = 24$ Multiply by 12: $4x - 24 = 3x$, so $x = 24$.

2. What is the solution set of 7(x + 1) = 7x + 7?

A. No solution, ∅
B. All real numbers
C. Only the value x = 0
D. Only the value x = −1

B. All real numbers Both sides are identical, so the variable cancels to a true statement.

3. Solve A = P + Prt for r.

A. r = Pt/(A − P)
B. r = A/(Pt) − P
C. r = (A − P)/(Pt)
D. r = (A + P)/(Pt)

C. r = (A − P)/(Pt) Subtract P: A − P = Prt. Divide by Pt, with P, t ≠ 0.

4. After a 30% discount, a coat costs $63. What was the original price?

A. $210
B. $81.90
C. $93
D. $90

D. $90 Keeping 70%: 0.70p = 63, so p = 90.

5. Solve $-5x + 1 < 16$.

A. $[-3, \infty)$

B. $(-\infty, -3)$

C. $(-3, \infty)$

D. $(3, \infty)$

C. $(-3, \infty)$ $-5x < 15$. Dividing by -5 flips the symbol: $x > -3$.

6. Solve $|2x - 1| \geq 7$.

A. $[-3, 4]$, the middle

B. $(-\infty, -3] \cup [4, \infty)$

C. $(-\infty, -4] \cup [3, \infty)$

D. $(-\infty, -3) \cup (4, \infty)$

B. $(-\infty, -3] \cup [4, \infty)$ Greater than gives an "or": $2x - 1 \leq -7$ or $2x - 1 \geq 7$.

CHAPTER 3

Graphs and Lines

Read it in full: **Chapter 3 in the textbook**. Practice it in the lab: **Parallel and Perpendicular** and **Build a Linear Model**.

What to know

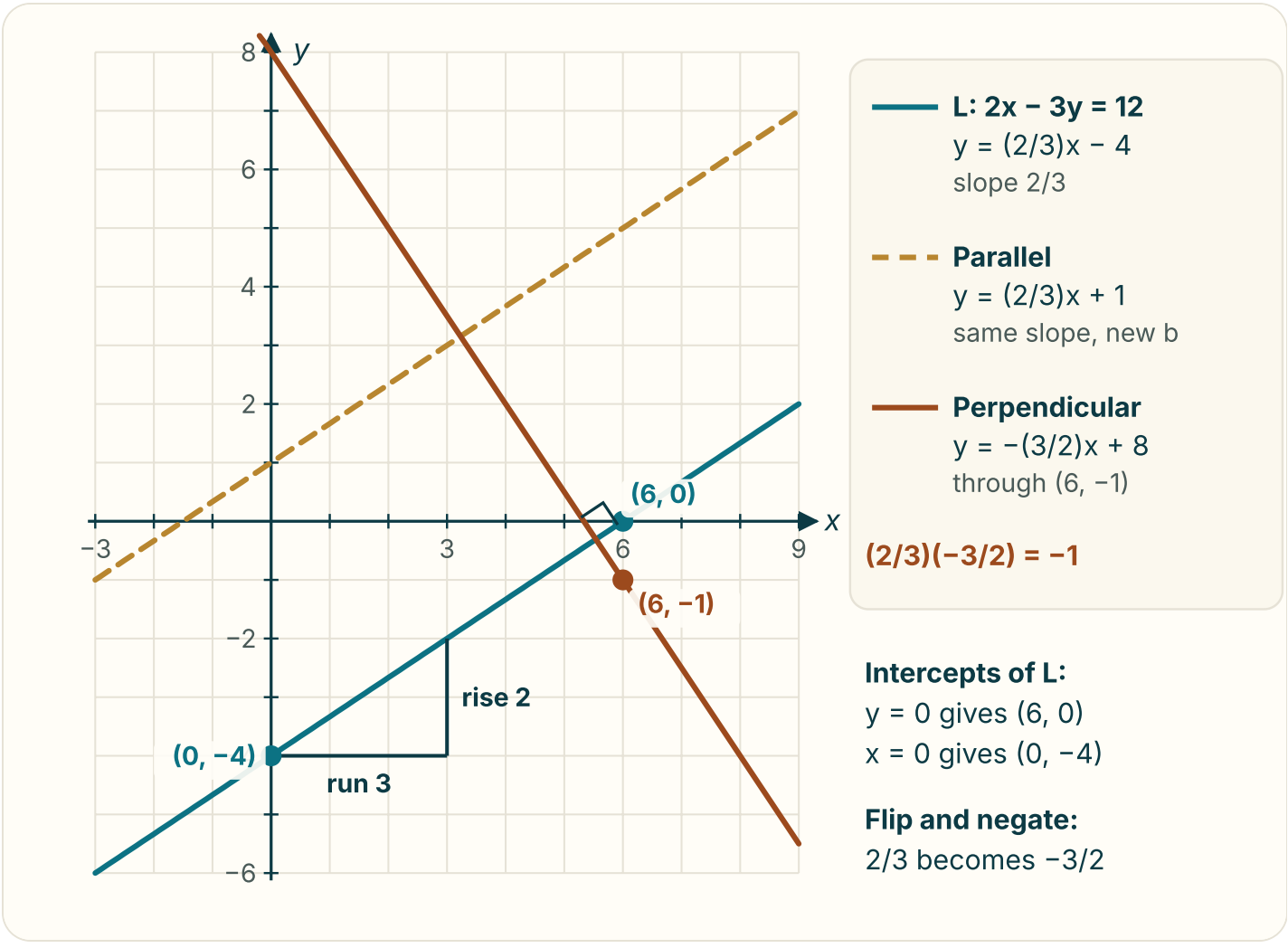
An ordered pair (x, y) is horizontal first. Quadrants run I to IV counterclockwise from the upper right, and points on an axis are in none. Set $y = 0$ for the x-intercept and $x = 0$ for the y-intercept, and report each as a point.

Slope $m = (y_2 - y_1)/(x_2 - x_1)$ is rise over run, a rate with y-units per x-unit. Zero slope is horizontal ($y = k$); undefined slope is vertical ($x = k$). The three forms are slope-intercept $y = mx + b$, point-slope $y - y_1 = m(x - x_1)$, and standard $Ax + By = C$.

Parallel lines share a slope with different intercepts. Perpendicular slopes multiply to −1: flip and negate. A linear model uses the slope as the rate and the intercept as the starting value; check it with the unused data point, and do not extrapolate far.

LOCK THESE IN

- Intercepts are points.
- Same subtraction order top and bottom.
- Zero slope ≠ undefined slope.
- Point-slope is the building form.
- Perpendicular: product −1.
- Slope is a rate with units.



Slope is rise over run, and it decides direction. Parallel lines share a slope with different intercepts. Perpendicular slopes multiply to −1: flip the fraction, then change the sign.

Chapter 3 formulas

Item	Formula or rule	When to use it
Slope	$(y_2 - y_1)/(x_2 - x_1)$	Two points
Point-slope	$y - y_1 = m(x - x_1)$	Building a line
Standard	$Ax + By = C$; $m = -A/B$	Integer form
Perpendicular	$m_1m_2 = -1$	Right angles

Worked examples

EXAMPLE 3.1 · TWO POINTS TO A LINE

Write the line through $(-3, 5)$ and $(1, -3)$.

STEP 1 · SLOPE

$(-3 - 5)/(1 + 3) = -8/4 = -2$.

STEP 2 · POINT-SLOPE

$y - 5 = -2(x + 3)$, so **$y = -2x - 1$**, or $2x + y = -1$.

CHECK THE OTHER POINT

$-2(1) - 1 = -3$.

EXAMPLE 3.2 · A PERPENDICULAR LINE

Find the line through $(3, 1)$ perpendicular to $3x - 4y = 8$.

STEP 1 · SLOPE OF THE GIVEN LINE

$y = (3/4)x - 2$, so $m = 3/4$.

STEP 2 · FLIP AND NEGATE

$m = -4/3$.

STEP 3 · POINT-SLOPE

$y - 1 = -(4/3)(x - 3)$, so **$y = -(4/3)x + 5$**.

TRAPS PEOPLE FALL INTO

- **Writing an intercept as a lone number.** Write (0, 4).
- **Mixing the subtraction order.** It flips the sign of m.
- **Flipping without negating.** 3/4 becomes −4/3.
- **Reading m from Ax + By = C.** Solve for y first.

Chapter 3 self-test · six items

1. Where does 4x + 5y = 20 cross the y-axis?

A. (0, 4)

B. (4, 0)

C. (0, 5)

D. (5, 0)

A. (0, 4) Set x = 0: 5y = 20, so y = 4.

2. What is the slope through (2, 7) and (6, −1)?

A. −1/2

B. 2

C. 3/2

D. −2

D. −2 (−1 − 7)/(6 − 2) = −8/4 = −2.

3. A line passes through (4, −3) and never rises or falls. What is its equation?

A. x = −3

B. x = 4

C. y = −3

D. y = 4

C. y = −3 A horizontal line keeps one y-value.

4. Rewrite 2x + 7y = 14 to read off its slope. What is it?

A. 2
B. −2/7
C. −7/2
D. 2/7

B. −2/7 y = −(2/7)x + 2.

5. Which line is parallel to y = 5x − 2 and passes through (1, 4)?

A. y = −(1/5)x + 4
B. y = 5x − 2
C. y = 5x + 4
D. y = 5x − 1

D. y = 5x − 1 Same slope 5: y − 4 = 5(x − 1).

6. A gym costs $100 for 3 months and $180 for 7 months. What is the one-time fee?

A. $100
B. $40
C. $20
D. $60

B. $40 Slope (180 − 100)/(7 − 3) = 20 a month. Then 100 − 3(20) = 40.

CHAPTER 4

Introduction to Functions

Read it in full: **Chapter 4 in the textbook**. Practice it in the lab: **The Function Machine** and **Reading Domain and Range**.

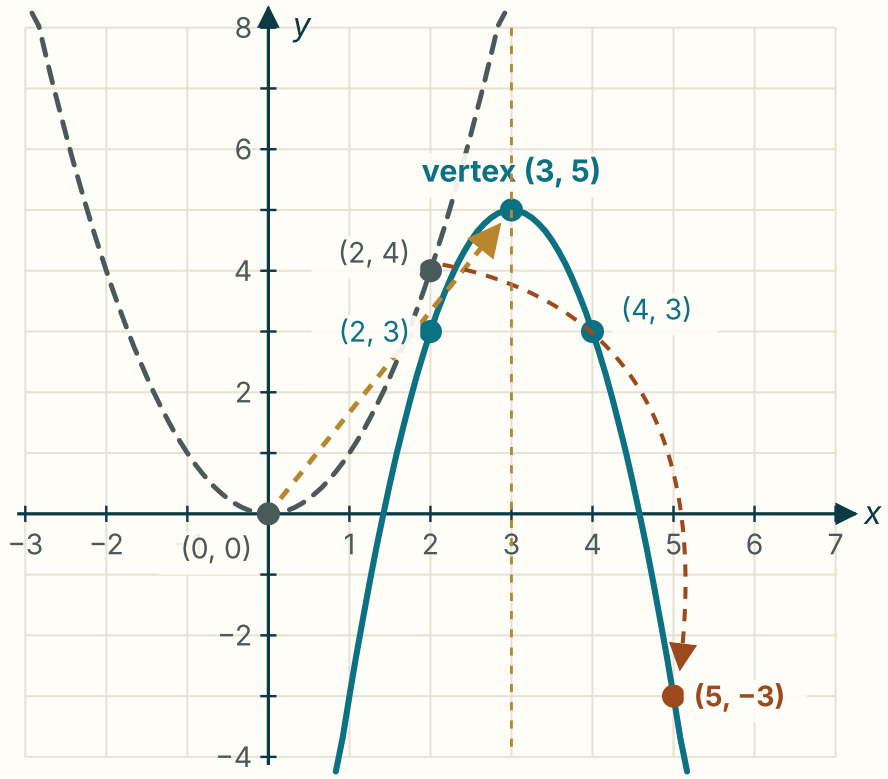
What to know

A function gives each input exactly one output; repeated outputs are fine. The vertical line test is that rule drawn. f(x) is "f of x," never multiplication: substitute the input in parentheses. f(0) is the y-intercept, and f(x) = 0 gives the x-intercepts.

Find the domain by exclusion: no zero denominators and no negatives under even roots. A root in a denominator needs a strict inequality. Read domain left to right and range bottom to top on a graph.

The toolkit is x, x^2, $|x|$, $\sqrt{x}$, x^3 and $1/x$. Outside changes are vertical and literal; inside changes are horizontal and reversed. In $a \cdot f(x - h) + k$, the point (x_0, y_0) moves to $(x_0 + h, a \cdot y_0 + k)$.

> **LOCK THESE IN**
>
> - One input, one output.
> - f(x) is not multiplication.
> - Domain by exclusion.
> - Bracket only an attained endpoint.
> - Inside is reversed.
> - Check transformations with a point.



- - - $f(x) = x^2$

——— $g(x) = -2(x - 3)^2 + 5$

1. x − 3 inside: right 3
2. 2 outside: stretch by 2
3. minus: flip over x-axis
4. +5 outside: up 5

Track (2, 4):

$x - 3 = 2$, so $x = 5$

$4(-2) + 5 = -3$

lands at (5, −3)

Range $(-\infty, 5]$

Inside changes are horizontal and reversed; outside changes are vertical and literal. The vertex moves from (0, 0) to (3, 5), and the point (2, 4) lands at (5, −3). Checking $g(5) = -3$ confirms it.

Chapter 4 rules

Item	Formula or rule	When to use it
Function test	No input with two outputs	Pairs, tables, graphs
Domain	Exclude zero denominators and negative radicands	Every formula
Transformation	$a \cdot f(x - h) + k$	Right h, stretch a, up k
Reflections	$-f(x)$: x-axis; $f(-x)$: y-axis	Minus outside or inside

Worked examples

EXAMPLE 4.1 · EVALUATING

Let $f(x) = x^2 - 3x + 2$. Find $f(-1)$ and $f(x + 2)$.

STEP 1 · NUMBER INPUT

$f(-1) = 1 + 3 + 2 = \mathbf{6}$.

STEP 2 · EXPRESSION INPUT

$(x + 2)^2 - 3(x + 2) + 2 = x^2 + 4x + 4 - 3x - 6 + 2 = \mathbf{x^2 + x}$.

CHECK WITH X = 1

$f(3) = 2$, and $1 + 1 = 2$.

EXAMPLE 4.2 · DOMAIN AND RANGE

Find the domain of $h(x) = 3/\sqrt{(x + 5)}$ and the range of $g(x) = -(x - 1)^2 + 4$.

STEP 1 · ROOT IN A DENOMINATOR

$x + 5 > 0$, so the domain is $\mathbf{(-5, \infty)}$.

STEP 2 · VERTEX

g has vertex (1, 4) and opens down, so the range is $\mathbf{(-\infty, 4]}$.

TRAPS PEOPLE FALL INTO

- **Rejecting repeated outputs.** Only a repeated input with two outputs fails.
- **Writing $(a + 1)^2 = a^2 + 1$.** It is $a^2 + 2a + 1$.
- **Excluding a numerator zero.** Only denominators matter.
- **Moving $f(x - 3)$ left.** It moves right.

Chapter 4 self-test · six items

1. Which set of ordered pairs is a function?

A. {(2, 3), (3, 3), (4, 5)}

B. {(1, 1), (1, 2), (1, 3)}

C. {(0, 5), (6, 2), (0, 7)}

D. {(2, 3), (2, 4), (5, 1)}

A. {(2, 3), (3, 3), (4, 5)} No input repeats. The others each repeat an input with a new output.

2. Let $f(x) = 2x^2 - x$. What is $f(-3)$?

A. 33
B. 15
C. 21
D. −21

C. 21 $2(9) - (-3) = 18 + 3 = 21$.

3. Which inputs must be excluded from $(x + 1)/(x^2 - 25)$?

A. $x > 5$ only
B. All reals except −1
C. All reals except 25
D. All reals except ±5

D. All reals except ±5 $x^2 - 25 = 0$ at $x = 5$ and $x = -5$.

4. What is the domain of $\sqrt{(3 - x)}$?

A. $[3, \infty)$
B. $(-\infty, 3]$
C. $(-3, \infty)$
D. $(-\infty, 3)$

B. $(-\infty, 3]$ $3 - x \geq 0$ gives $x \leq 3$, and $\sqrt{0}$ is allowed.

5. Where is the corner of $y = |x + 2| - 1$?

A. $(-2, -1)$
B. $(-2, 1)$
C. $(1, -2)$
D. $(2, -1)$

A. $(-2, -1)$ Inside +2 moves left 2; outside −1 moves down 1.

6. (9, 3) is on $y = \sqrt{x}$. Where does it land on $y = -2\sqrt{x}$?

A. $(18, -6)$
B. $(9, -3)$
C. $(9, -6)$
D. $(-9, 3)$

C. $(9, -6)$ Outside changes keep x: $3(-2) = -6$.

CHAPTER 5

Systems of Equations and Inequalities

Read it in full: **Chapter 5 in the textbook**. Practice it in the lab: **Substitution, Step by Step** and **Where the Shading Overlaps**.

What to know

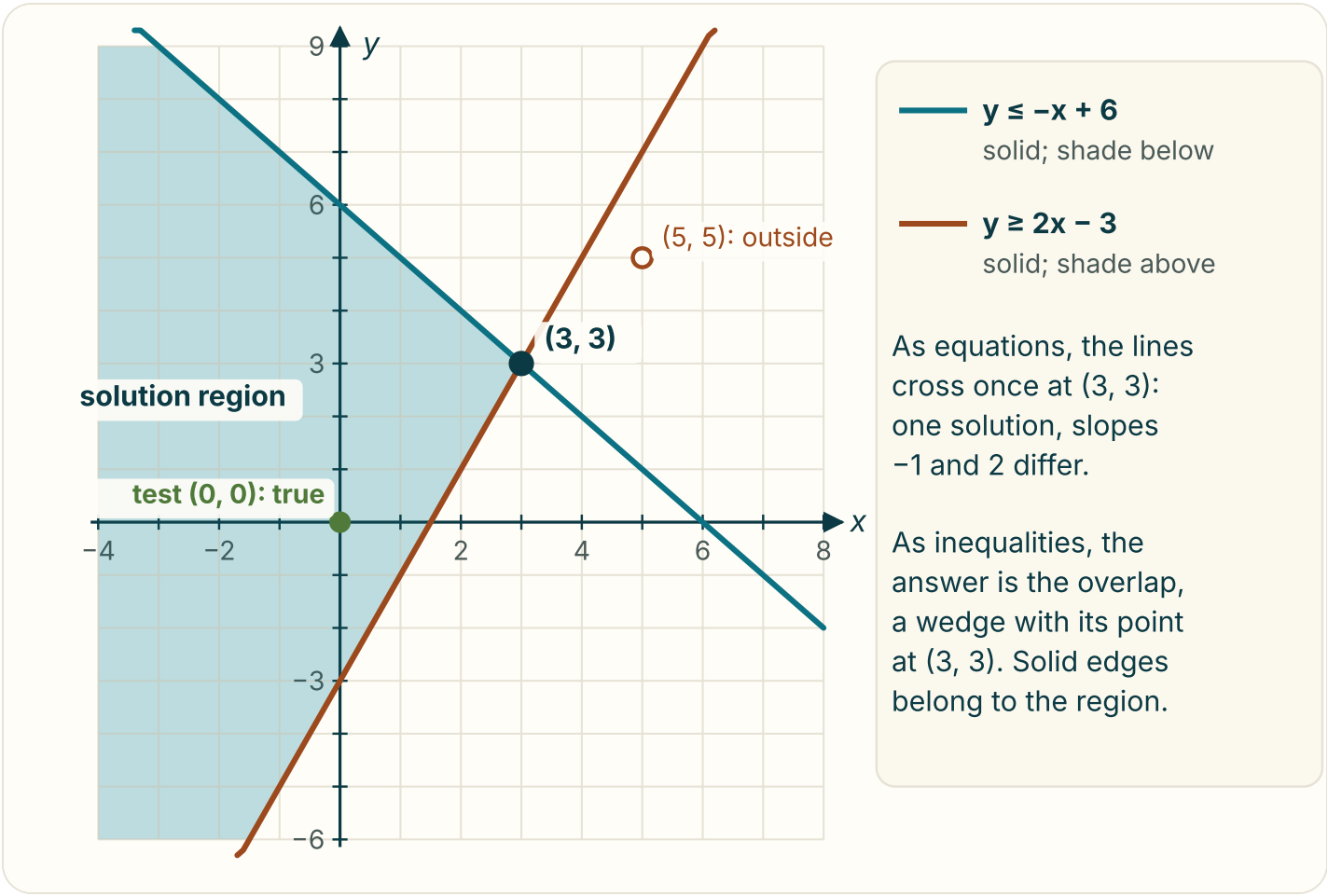
A solution of a system satisfies every equation. Two lines give one solution (different slopes), none (parallel), or infinitely many (same line). Substitution isolates a variable and swaps it into the other equation; elimination multiplies whole equations so a variable cancels when added.

A false ending such as $0 = 5$ means no solution; a true ending such as $0 = 0$ means infinitely many. A 3×3 system is reduced by eliminating the same variable from two pairs, then back-substituting, using every equation.

Mixture problems pair an amount equation with a substance equation; current problems use b + c and b − c. For inequalities, draw solid lines for ≤ and ≥, dashed for < and >, test a point, and keep the overlap.

LOCK THESE IN

- Check in every equation.
- Compare slopes first.
- Multiply whole equations.
- 0 = 0: infinitely many; 0 = nonzero: none.
- Eliminate the same variable twice.
- Track the substance in mixtures.



One crossing point for equations; one shaded overlap for inequalities. Different slopes guarantee exactly one intersection. For the inequalities, a test point picks each side, and the answer is only where both shadings agree.

Chapter 5 tools

Item	Formula or rule	When to use it
Substitution	Isolate, substitute, solve, back-substitute	A coefficient of ±1
Elimination	Make opposite coefficients, add	Ax + By = C
Mixture	$p_1x + p_2y = p_{mix}(x + y)$	Solutions
Inequality graph	Solid ≤ ≥, dashed < >; test point	Shading

Worked examples

EXAMPLE 5.1 · SUBSTITUTION

Solve $x = 2y - 1$ and $3x + y = 18$.

STEP 1 · SUBSTITUTE

$3(2y - 1) + y = 18$, so $7y - 3 = 18$ and $y = 3$.

STEP 2 · BACK-SUBSTITUTE

$x = 2(3) - 1 = 5$. The solution is **(5, 3)**.

CHECK

$15 + 3 = 18$.

EXAMPLE 5.2 · ELIMINATION

Solve $5x + 2y = 4$ and $3x - 4y = 18$.

STEP 1 · MULTIPLY THE FIRST BY 2

$10x + 4y = 8$.

STEP 2 · ADD

$13x = 26$, so $x = 2$.

STEP 3 · BACK-SUBSTITUTE

$10 + 2y = 4$, so $y = -3$. The solution is **(2, −3)**.

TRAPS PEOPLE FALL INTO

- **Checking one equation.** Check them all.
- **Multiplying only part of an equation.** The constant must be multiplied too.
- **Tracking liquid instead of substance.** The second equation counts the acid or salt.
- **Forgetting the flip in inequalities.** Dividing by a negative flips the symbol.

Chapter 5 self-test · six items

1. How many solutions does y = 2x − 3, y = 2x + 4 have?

A. None
B. Two
C. Infinitely many
D. One

A. None Same slope, different intercepts: parallel lines.

2. Add the equations 2x + y = 7 and x − y = 2. What solution results?

A. (1, 3)
B. (3, −1)
C. (2, 3)
D. (3, 1)

D. (3, 1) Adding gives 3x = 9, so x = 3 and y = 1.

3. Solve z = 2, y − z = 1, x + y + z = 6.

A. (2, 3, 1)
B. (1, 2, 3)
C. (1, 3, 2)
D. (3, 1, 2)

C. (1, 3, 2) z = 2, so y = 3, so x = 6 − 3 − 2 = 1.

4. How many liters of 30% acid are mixed with 10% acid to make 20 L of 25% acid?

A. 5 L
B. 15 L
C. 12.5 L
D. 10 L

B. 15 L $x + y = 20$ and $0.1x + 0.3y = 5$ give $y = 15$.

5. A plane flies 600 miles in 2 hours with the wind and 3 hours against it. What is the wind speed?

A. 50 mi/h
B. 25 mi/h
C. 100 mi/h
D. 250 mi/h

A. 50 mi/h $p + w = 300$ and $p - w = 200$, so $w = 50$.

6. How do you graph $y \leq -x + 3$?

A. Solid line, shade away from (0, 0)
B. Dashed line, shade away from (0, 0)
C. Solid line, shade toward (0, 0)
D. Dashed line, shade toward (0, 0)

C. Solid line, shade toward (0, 0) $\leq$ means solid. Testing (0, 0): $0 \leq 3$ is true.

CHAPTER 6

Polynomials and Factoring

Read it in full: **Chapter 6 in the textbook**. Practice it in the lab: **Undoing the Rectangle** and **Special Patterns**.

What to know

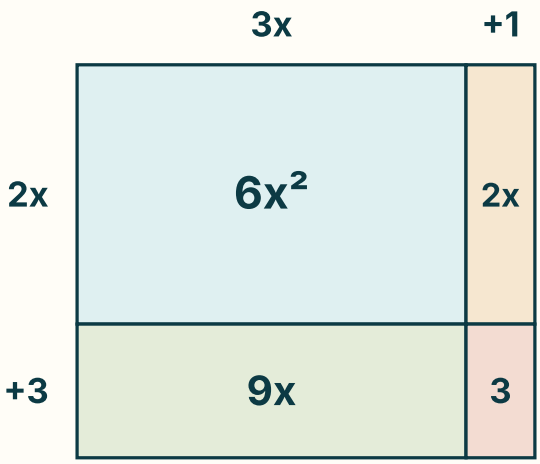
A polynomial has whole-number exponents with no variable in a denominator, radical or exponent. Degree is the largest exponent (or sum of exponents in a term). Combine only like terms, and a subtracted group flips every sign.

Multiply every term by every term; FOIL is the two-binomial case. $(a \pm b)^2 = a^2 \pm 2ab + b^2$ and $(a + b)(a - b) = a^2 - b^2$. Always pull out the GCF first, then count terms.

Four terms: group. $x^2 + bx + c$: product c, sum b. $ax^2 + bx + c$: the AC method. Difference of squares factors; a sum of squares is prime. Cubes use SOAP. Check every factorization by multiplying back.

LOCK THESE IN

- GCF first.
- Subtracting flips every sign in the group.
- $(a + b)^2$ has a 2ab term.
- AC: product ac, sum b.
- $a^2 + b^2$ is prime.
- SOAP for cubes.



Area = (2x + 3)(3x + 1)

= 6x² + 9x + 2x + 3

= 6x² + 11x + 3

AC method for 6x² + 11x + 3

1. a · c = 6 · 3 = 18
2. product 18, sum 11:

1 + 18 = 19, 2 + 9 = 11

3. split: 6x² + 9x + 2x + 3
4. group: 3x(2x + 3) + 1(2x + 3)
5. **factor: (2x + 3)(3x + 1)**
6. check by FOIL:

6x² + 2x + 9x + 3

The two middle boxes are 9x and 2x.

Factoring is the rectangle run backward. The corner boxes are fixed by a and c, and the two middle boxes must multiply to a · c = 18 and add to b = 11. That is the whole AC method.

Chapter 6 patterns

Item	Formula or rule	When to use it
Difference of squares	$(a + b)(a - b)$	$a^2 - b^2$
Perfect square	$(a \pm b)^2$	Middle term 2ab
Sum of cubes	$(a + b)(a^2 - ab + b^2)$	$a^3 + b^3$
Difference of cubes	$(a - b)(a^2 + ab + b^2)$	$a^3 - b^3$

Worked examples

EXAMPLE 6.1 · MULTIPLYING

Multiply $(3x - 2)(x^2 + x - 4)$.

STEP 1 · DISTRIBUTE 3X

$3x^3 + 3x^2 - 12x$.

STEP 2 · DISTRIBUTE −2

$-2x^2 - 2x + 8$.

STEP 3 · COMBINE

$3x^3 + x^2 - 14x + 8$. At x = 1 both forms give −2.

EXAMPLE 6.2 · FACTORING COMPLETELY

Factor $3x^3 - 3x^2 - 36x$.

STEP 1 · GCF

$3x(x^2 - x - 12)$.

STEP 2 · TRINOMIAL

Product −12, sum −1: −4 and 3.

STEP 3 · ANSWER

$3x(x - 4)(x + 3)$.

TRAPS PEOPLE FALL INTO

- **Adding exponents when adding terms.** $3x^2 + 4x^2 = 7x^2$.
- **Skipping the GCF.** It shrinks every later step.
- **Factoring $x^2 + 25$.** A sum of squares is prime.
- **Stopping early.** Factor until nothing breaks down.

Chapter 6 self-test · six items

1. The polynomial $4x^2y^3 - 7x^4 + 1$ has what degree?

A. 4

B. 3

C. 5

D. 9

C. 5 The first term has degree 2 + 3 = 5, the largest.

2. Write $(x - 5)^2$ as a trinomial.

A. $x^2 - 10x + 25$

B. $x^2 + 10x + 25$

C. $x^2 - 10x - 25$

D. $x^2 - 5x + 25$

A. $x^2 - 10x + 25$ $a^2 - 2ab + b^2$ with $a = x$ and $b = 5$.

3. Group the terms of $x^3 - 4x^2 + 5x - 20$ and factor.

A. $(x - 4)(x + 5)$

B. $(x - 4)(x^2 + 5)$

C. $(x + 4)(x^2 - 5)$

D. $(x - 5)(x^2 + 4)$

B. $(x - 4)(x^2 + 5)$ $x^2(x - 4) + 5(x - 4)$.

4. Use the AC method on $3x^2 + 10x + 8$.

A. $(3x + 2)(x + 4)$

B. $(3x + 8)(x + 1)$

C. $(x + 4)(x + 2)$

D. $(3x + 4)(x + 2)$

D. $(3x + 4)(x + 2)$ $ac = 24$; product 24, sum 10: 6 and 4. Then $3x(x + 2) + 4(x + 2)$.

5. Factor $9x^2 - 49$.

A. $(3x + 7)(3x - 7)$

B. $(9x + 7)(x - 7)$

C. $(3x - 7)(3x - 7)$

D. $(3x + 7)(3x + 7)$

A. $(3x + 7)(3x - 7)$ Difference of squares with a = 3x and b = 7.

6. Factor the sum of cubes $8x^3 + 1$.

A. $(2x - 1)(4x^2 + 2x + 1)$

B. $(2x + 1)(4x^2 + 2x + 1)$

C. $(2x + 1)(4x^2 + 1)$

D. $(2x + 1)(4x^2 - 2x + 1)$

D. $(2x + 1)(4x^2 - 2x + 1)$ SOAP with a = 2x and b = 1.

CHAPTER 7

Rational Expressions

Read it in full: **Chapter 7 in the textbook**. Practice it in the lab: **The Bottoms Must Match** and **Two Roads, One Answer**.

What to know

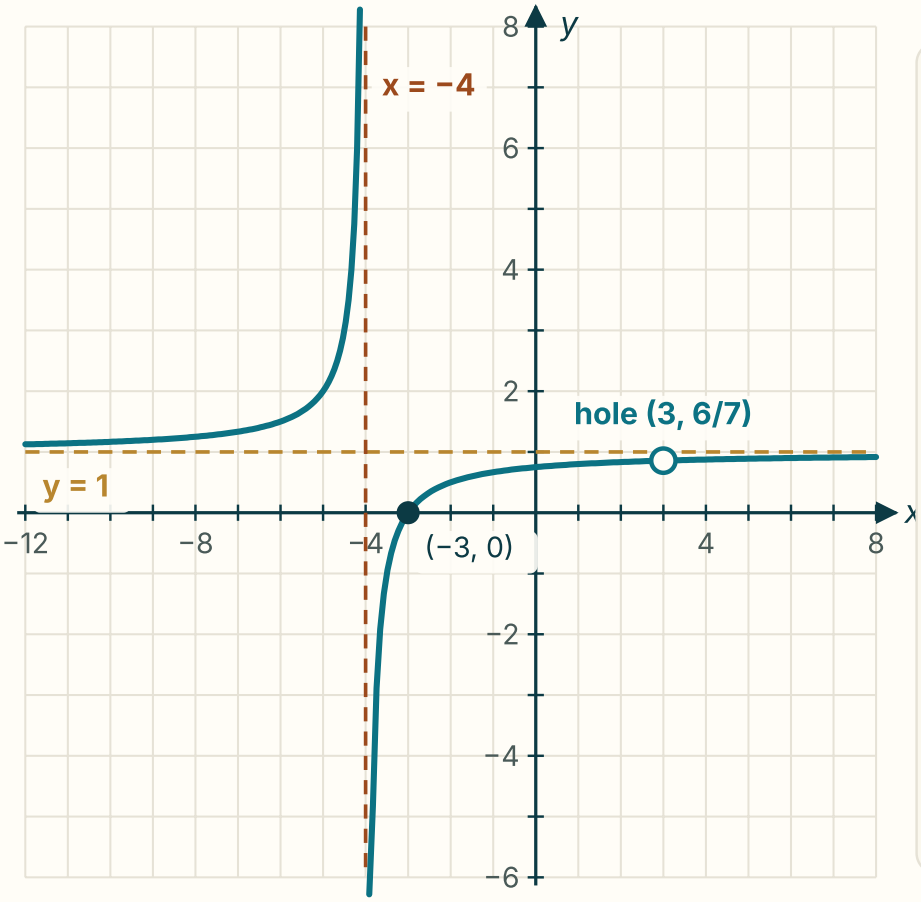
A rational expression is a polynomial over a polynomial. List the excluded values from the original denominators first; they survive canceling. Factor, then cancel factors, never terms. $(b - a)/(a - b) = -1$.

Multiply after factoring and canceling. Divide by flipping the divisor, which also excludes its numerator's zeros. Add and subtract over the LCD: each distinct factor at its highest power, and parenthesize the second numerator when subtracting.

A complex fraction is a division; multiply top and bottom by the inner LCD. A rational equation is cleared by the LCD, solved, then screened: excluded candidates are extraneous, and if all are, there is no solution.

LOCK THESE IN

- Excluded values first.
- Cancel factors only.
- Divisor numerator is excluded too.
- LCD: highest power of each factor.
- Parenthesize when subtracting.
- Screen every candidate.



f(x) =

$\frac{(x^2 - 9)}{(x^2 + x - 12)}$

factor:

$\frac{(x + 3)(x - 3)}{(x + 4)(x - 3)}$

simplified:

$\frac{(x + 3)}{(x + 4)}$

Excluded: $x \neq -4, 3$

$x = 3$: the factor canceled, so the graph has a hole.

$x = -4$: still in the denominator, so a vertical asymptote.

Excluded values survive simplifying. The simplified form $(x + 3)/(x + 4)$ looks fine at $x = 3$, but the original is undefined there, so the graph has a hole. A factor that stays in the denominator gives an asymptote instead.

Chapter 7 rules

Item	Formula or rule	When to use it
Simplify	Factor, cancel factors	Keep restrictions
Divide	$(a/b) \div (c/d) = (a/b)(d/c)$	$c \neq 0$
LCD	Distinct factors, highest powers	Adding, subtracting, equations
Work rate	$1/t_1 + 1/t_2 = 1/t$	Two workers

Worked examples

EXAMPLE 7.1 · SIMPLIFYING

Simplify $(x^2 + 3x - 10)/(x^2 - 4)$.

STEP 1 · FACTOR

$[(x + 5)(x - 2)]/[(x - 2)(x + 2)]$, so $x \neq \pm 2$.

STEP 2 · CANCEL

$(x + 5)/(x + 2)$, with $x \neq 2, -2$.

CHECK AT 0

$-10/-4 = 2.5$ and $5/2 = 2.5$.

EXAMPLE 7.2 · A RATIONAL EQUATION

Solve $1/x + 1/(x + 2) = 4/[x(x + 2)]$.

STEP 1 · EXCLUDE

$x \neq 0$ and $x \neq -2$.

STEP 2 · CLEAR WITH THE LCD X(X + 2)

$(x + 2) + x = 4$, so $2x = 2$ and **$x = 1$**.

VERIFY

$1 + 1/3 = 4/3$, and $4/(1 \cdot 3) = 4/3$.

TRAPS PEOPLE FALL INTO

- **Canceling terms.** (x + 4)/4 is not x.
- **Restrictions found after canceling.** Use the original denominators.
- **x − 2x − 6 for x − (2x − 6).** Distribute the minus.
- **Keeping an excluded candidate.** Discard it.

Chapter 7 self-test · six items

1. What are the excluded values of $5/(x^2 - 7x + 12)$?

A. $x \neq 5$ only

B. $x \neq 3$ and $x \neq 4$

C. $x \neq 12$ only

D. $x \neq -3$ and $x \neq -4$

B. $x \neq 3$ and $x \neq 4$ $x^2 - 7x + 12 = (x - 3)(x - 4)$.

2. Simplify $(x - 4)/(4 - x)$.

A. −1

B. 1

C. 0

D. $x - 4$

A. −1 $4 - x = -1(x - 4)$, with $x \neq 4$.

3. Simplify $(x^2 - 1)/(x + 3) \div (x - 1)/(x^2 + 3x)$.

A. $x + 1$

B. $(x + 1)/x$

C. $x(x - 1)$

D. $x(x + 1)$

D. x(x + 1) Flip, factor and cancel (x − 1) and (x + 3).

4. Write $3/(x - 2) - 1/(x + 2)$ as one fraction.

A. $(2x + 4)/[(x - 2)(x + 2)]$
B. $(2x - 4)/[(x - 2)(x + 2)]$
C. $(2x + 8)/[(x - 2)(x + 2)]$
D. $2/[(x - 2)(x + 2)]$

C. $(2x + 8)/[(x - 2)(x + 2)]$ $3(x + 2) - (x - 2) = 2x + 8$.

5. Solve $x^2/(x - 5) = 25/(x - 5)$.

A. No solution
B. $x = 5$ only
C. $x = 5$ or $x = -5$
D. $x = -5$ only

D. x = −5 only $x^2 = 25$, but $x = 5$ is excluded, so it is extraneous.

6. Pipes fill a tank in 3 hours and 6 hours. Together, how long?

A. 9 hours
B. 4.5 hours
C. 2 hours
D. 1.5 hours

C. 2 hours $1/3 + 1/6 = 1/2$, so $t = 2$.

CHAPTER 8

Radicals and Rational Exponents

Read it in full: **Chapter 8 in the textbook**. Practice it in the lab: **The Radical Wears a Fraction** and **Clearing the Radical from the Bottom**.

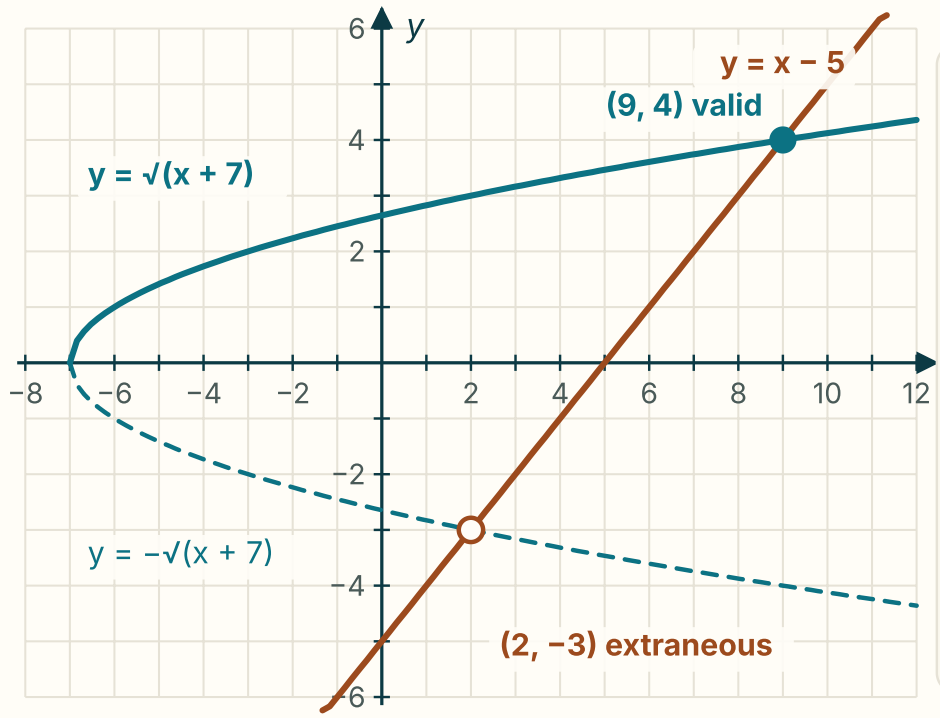
What to know

√ returns the principal root, and $\sqrt{(x^2)} = |x|$. Even roots of negatives are not real; odd roots are. Simplify with the largest perfect power matching the index. Only like radicals combine, and $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$.

Multiply radicands, then simplify. Rationalize one-term denominators with the radical itself and two-term denominators with the conjugate. $a^{m/n} = (\sqrt[n]{a})^m$: root first; a negative exponent means reciprocal.

To solve a radical equation, isolate the radical, raise both sides to the index, solve, and check every candidate. Squaring can invent extraneous solutions; an isolated root equal to a negative has no solution.

> **LOCK THESE IN**
>
> - $\sqrt{25} = 5$, not ±5.
> - Simplify before combining.
> - Conjugate: flip only the middle sign.
> - Denominator is the root.
> - Negative exponent: reciprocal.
> - Check every radical-equation answer.



Solve √(x + 7) = x − 5

square both sides:

$x + 7 = x^2 - 10x + 25$

$0 = (x - 2)(x - 9)$

check x = 9: 4 = 4

check x = 2: 3 ≠ −3

Squaring also solves the dashed mirror equation, so x = 2 rides along.

Squaring cannot tell a root from its mirror. Both branches square to the same equation, so the algebra returns x = 2 and x = 9. Only the check in the original keeps x = 9 and rejects x = 2.

Chapter 8 rules

Item	Formula or rule	When to use it
Product	$\sqrt{(ab)} = \sqrt{a}\sqrt{b}$	Simplifying
Conjugate	$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$	Rationalizing
Rational exponent	$a^{m/n} = (\sqrt[n]{a})^m$	Converting
Equation	Isolate, raise, solve, check	Variable under a root

Worked examples

EXAMPLE 8.1 · SIMPLIFYING AND COMBINING

Simplify $\sqrt{(72x^3)}$ for $x \geq 0$, and $3\sqrt{8} + \sqrt{50}$.

STEP 1 · PERFECT PARTS

$72x^3 = 36x^2 \cdot 2x$, so **$6x\sqrt{(2x)}$**.

STEP 2 · LIKE RADICALS

$3(2\sqrt{2}) + 5\sqrt{2} = \mathbf{11\sqrt{2}}$.

EXAMPLE 8.2 · A RADICAL EQUATION

Solve $\sqrt{(x + 3)} + 3 = x$.

STEP 1 · ISOLATE

$\sqrt{(x + 3)} = x - 3$.

STEP 2 · SQUARE AND SOLVE

$x + 3 = x^2 - 6x + 9$, so $(x - 1)(x - 6) = 0$.

STEP 3 · CHECK

$x = 6$: $3 + 3 = 6$, valid. $x = 1$: $2 + 3 = 5 \neq 1$, extraneous. Answer **$x = 6$**.

TRAPS PEOPLE FALL INTO

- **Adding unlike radicals.** $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$.
- **Squaring before isolating.** Isolate the root first.
- **Swapping terms in a conjugate.** Change only the middle sign.
- **Skipping the check.** Squaring invents answers.

Chapter 8 self-test · six items

1. Which is a real number?

A. $\sqrt{(-1)}$

B. $\sqrt[4]{(-81)}$

C. $\sqrt{(-36)}$

D. $\sqrt[5]{(-32)}$

D. $\sqrt[5]{(-32)}$ An odd index allows a negative radicand: the value is -2.

2. Simplify $\sqrt{75}$.

A. $25\sqrt{3}$
B. $5\sqrt{3}$
C. $15\sqrt{5}$
D. $3\sqrt{5}$

B. $5\sqrt{3}$ $75 = 25 \cdot 3$.

3. Multiply $\sqrt{14} \cdot \sqrt{21}$ and simplify.

A. $7\sqrt{6}$
B. $6\sqrt{7}$
C. $\sqrt{35}$
D. $49\sqrt{6}$

A. $7\sqrt{6}$ $\sqrt{294} = \sqrt{(49 \cdot 6)} = 7\sqrt{6}$.

4. Use the conjugate to clear the denominator of $3/(2 - \sqrt{3})$.

A. $(6 + 3\sqrt{3})/7$
B. $2 + \sqrt{3}$
C. $6 + 3\sqrt{3}$
D. $6 - 3\sqrt{3}$

C. $6 + 3\sqrt{3}$ Multiply by $2 + \sqrt{3}$: the bottom becomes $4 - 3 = 1$.

5. Evaluate $125^{2/3}$.

A. 1/25

B. 25

C. 5

D. 83.3

B. 25 $\sqrt[3]{125} = 5$, then $5^2 = 25$.

6. Write $x^{1/3} \cdot x^{1/2}$ as one power of x.

A. $x^{5/6}$

B. $x^{1/6}$

C. $x^{1/5}$

D. $x^{2/5}$

A. $x^{5/6}$ 1/3 + 1/2 = 2/6 + 3/6 = 5/6.

CHAPTER 9

Quadratic Equations and Functions

Read it in full: **Chapter 9 in the textbook**. Practice it in the lab: **Graph a Parabola** and **Projectile Motion**.

What to know

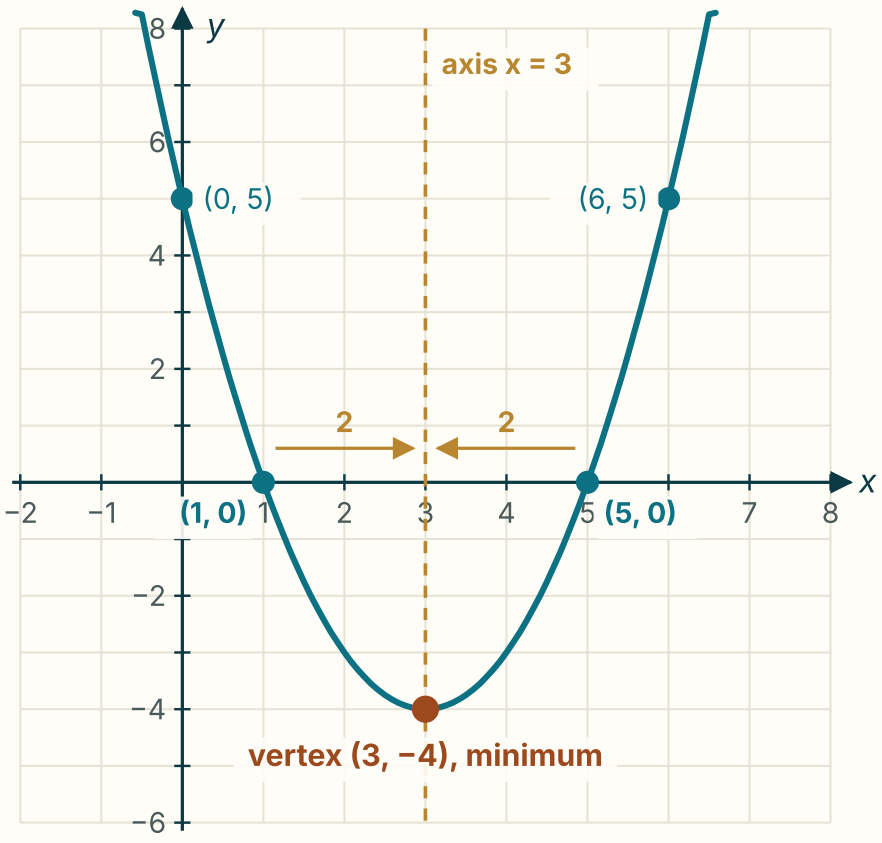
Put a quadratic in standard form $ax^2 + bx + c = 0$. Factor and use the zero product property, which works only against 0. Never divide by x. The square root property gives $\pm\sqrt{k}$.

Completing the square adds $(b/2)^2$ to both sides after making a = 1. The quadratic formula $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$ always works, and the discriminant counts real solutions: positive two, zero one, negative none.

A parabola opens up if $a > 0$ and down if $a < 0$. The axis is $x = -b/(2a)$, halfway between the roots, and vertex form $a(x - h)^2 + k$ shows the vertex. Applications are vertex questions or equation questions; reject impossible answers.

LOCK THESE IN

- Zero product property needs 0.
- ± with square roots.
- Add $(b/2)^2$ to both sides.
- Whole numerator over 2a.
- Axis $x = -b/(2a)$.
- Reject negative time or length.



$f(x) = x^2 - 6x + 5$

$a = 1 > 0$: opens up

$x = -b/(2a) = 6/2 = 3$

$f(3) = 9 - 18 + 5 = -4$

roots: $(x - 1)(x - 5)$

average of roots = 3

vertex form:

$(x - 3)^2 - 4$

discriminant:

$36 - 20 = 16 > 0$

two real roots

Vertex, axis, intercepts. The axis $x = -b/(2a) = 3$ sits halfway between the roots 1 and 5, a free check. Every point has a mirror image across it, such as (0, 5) and (6, 5).

Chapter 9 formulas

Item	Formula or rule	When to use it
Quadratic formula	$x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$	Any quadratic
Discriminant	$b^2 - 4ac$	Count real roots
Vertex form	$a(x - h)^2 + k$	Vertex (h, k)
Projectile	$-16t^2 + v_0t + h_0$	Feet and seconds

Worked examples

EXAMPLE 9.1 · THE FORMULA WITH A RADICAL ANSWER

Solve $2x^2 + 4x - 5 = 0$.

STEP 1 · DISCRIMINANT

$16 + 40 = 56$.

STEP 2 · FORMULA

$x = (-4 \pm \sqrt{56})/4 = (-4 \pm 2\sqrt{14})/4 = \mathbf{(-2 \pm \sqrt{14})/2}$.

STEP 3 · DECIMALS

About 0.871 and −2.871.

EXAMPLE 9.2 · VERTEX FORM

Write $f(x) = 2x^2 - 12x + 13$ in vertex form.

STEP 1 · FACTOR 2 FROM THE X TERMS

$2(x^2 - 6x) + 13$.

STEP 2 · COMPLETE THE SQUARE

$2(x^2 - 6x + 9) - 18 + 13$.

STEP 3 · ANSWER

$\mathbf{2(x - 3)^2 - 5}$, vertex (3, −5), minimum −5.

TRAPS PEOPLE FALL INTO

- **Using the zero product property on 18.** Rearrange to 0 first.
- **Dividing by x.** You lose x = 0.
- **Reading −b as negative.** If b = −5, then −b = 5.
- **Keeping negative time.** Reject it.

Chapter 9 self-test · six items

1. Solve $x^2 - 9x + 20 = 0$.

A. x = 4 or x = 5

B. x = −4 or x = −5

C. x = −4 or x = 5

D. x = 2 or x = 10

A. x = 4 or x = 5 (x − 4)(x − 5) = 0.

2. Find the discriminant of $4x^2 - 20x + 25 = 0$. How many real roots are there?

A. Three

B. Two

C. One

D. None

C. One 400 − 4(4)(25) = 0: a double root at 5/2.

3. Which constant turns $x^2 - 14x$ into a perfect square trinomial?

A. 7

B. 196

C. −49

D. 49

D. 49 $(-14/2)^2 = (-7)^2 = 49$.

4. Locate the turning point of $y = -x^2 + 6x - 4$.

A. (6, −4)

B. (3, 5)

C. (−3, −31)

D. (3, −13)

B. (3, 5) x = −6/(−2) = 3, and −9 + 18 − 4 = 5.

5. A rocket follows $h(t) = -16t^2 + 64t$. When does it land?

A. t = 2 s

B. t = 16 s

C. t = 64 s

D. t = 4 s

D. t = 4 s −16t(t − 4) = 0 gives t = 0 (launch) or t = 4.

6. Use the square root property on $x^2 = 50$.

A. $x = \pm 2\sqrt{5}$

B. $x = \pm 5\sqrt{2}$

C. $x = \pm 25$

D. $x = 5\sqrt{2}$ only

B. $x = \pm 5\sqrt{2}$ $x = \pm\sqrt{50} = \pm 5\sqrt{2}$.

CHAPTER 10

Polynomial and Rational Functions

Read it in full: **Chapter 10 in the textbook**. Practice it in the lab: **Synthetic Division** and **Graphing a Rational Function**.

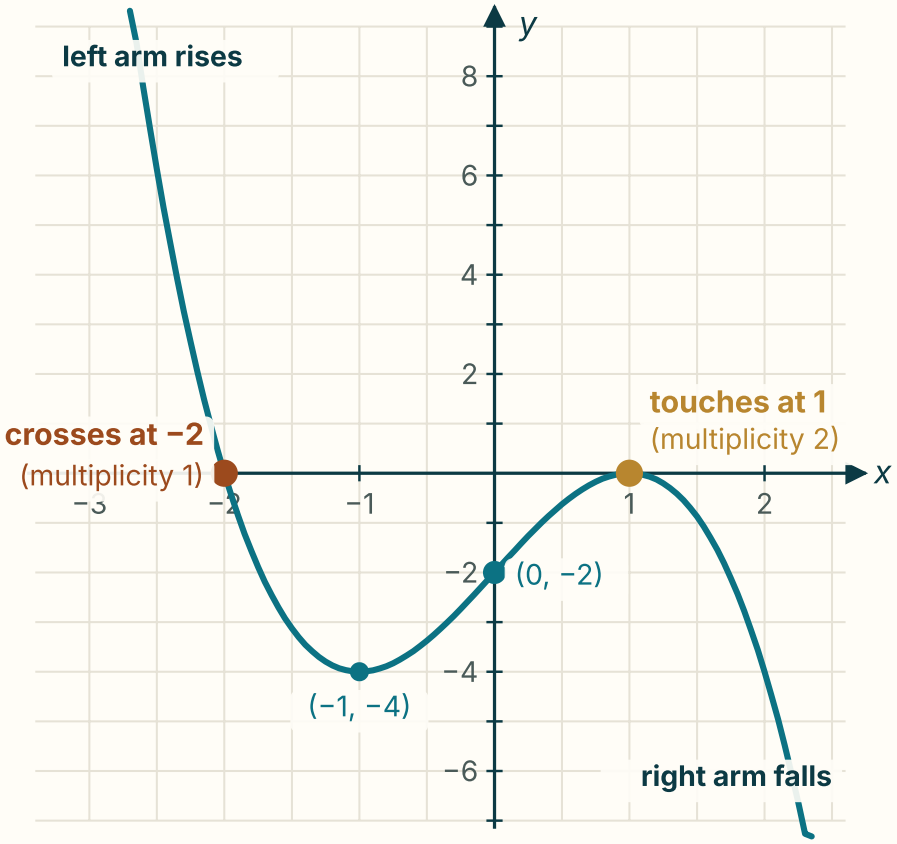
What to know

Degree and leading coefficient fix end behavior: even degree ends agree, odd degree ends disagree, a negative leading coefficient flips. Degree n allows at most n zeros and n − 1 turns. Odd multiplicity crosses; even multiplicity touches and turns.

Synthetic division divides by x − c: bring down, multiply, add, with 0 placeholders. The remainder equals P(c), and remainder 0 means (x − c) is a factor.

For rational functions, state the domain before canceling. A canceled factor leaves a hole; a surviving denominator factor gives a vertical asymptote. Compare degrees for the horizontal asymptote: smaller top gives y = 0, equal degrees give the ratio, top one higher gives a slant line.

> **LOCK THESE IN**
>
> - Even agrees, odd disagrees.
> - Odd crosses, even touches.
> - c from x − c.
> - Remainder = P(c).
> - Domain before canceling.
> - Hole vs asymptote by canceling.



$f(x) = -(x + 2)(x - 1)^2$

degree 1 + 2 = 3, odd

leading coefficient −1

odd and negative:

up on the left,

down on the right

odd multiplicity: cross

even multiplicity: touch

and turn back

$f(0) = -(2)(1) = -2$

at most 2 turning points

Two numbers fix the ends; multiplicity fixes each zero. Odd degree with a negative leading coefficient rises on the left and falls on the right. The graph crosses at the single zero and bounces at the double zero.

Chapter 10 rules

Item	Formula or rule	When to use it
End behavior	Degree parity and sign of a_n	Sketching
Factor Theorem	$P(c) = 0 \Leftrightarrow (x - c)$ is a factor	Peeling factors
Vertical asymptote	Surviving denominator zero	After canceling
Horizontal asymptote	$n < m$: 0; $n = m$: ratio	Far ends

Worked examples

EXAMPLE 10.1 · READING A FACTORED POLYNOMIAL

Describe $f(x) = x(x - 3)^2(x + 2)$.

STEP 1 · DEGREE AND SIGN

1 + 2 + 1 = 4, even, leading coefficient +1: both ends rise.

STEP 2 · ZEROS

0 and −2 cross (multiplicity 1); 3 touches and turns (multiplicity 2).

EXAMPLE 10.2 · A RATIONAL FUNCTION

Analyze $f(x) = (2x^2 - 8)/(x^2 - 3x + 2)$.

STEP 1 · FACTOR

$[2(x - 2)(x + 2)]/[(x - 1)(x - 2)]$; domain $x \neq 1, 2$.

STEP 2 · HOLE

Cancel $(x - 2)$: $f = 2(x + 2)/(x - 1)$; height $2(4)/1 = 8$, so **hole at (2, 8)**.

STEP 3 · ASYMPTOTES AND INTERCEPTS

VA $x = 1$; HA $y = 2$; x-intercept $(-2, 0)$; y-intercept $(0, -4)$.

TRAPS PEOPLE FALL INTO

- **Using c = 2 for x + 2.** c = −2.
- **Dropping placeholders.** Write 0 for missing powers.
- **Canceling before the domain.** The hole vanishes from your work.
- **Calling every excluded value an asymptote.** Canceled factors give holes.

Chapter 10 self-test · six items

1. Which way do the arms of $5x^3 - x$ point?

A. Both arms point up

B. Falls left, rises right

C. Both arms point down

D. Rises left, falls right

B. Falls left, rises right Odd degree, positive leading coefficient.

2. What are the zeros of $x^3 - 4x^2 - 5x$?

A. 0, 5 and −1

B. 5 and −1 only

C. 0 and 4

D. 0, −5 and 1

A. 0, 5 and −1 $x(x - 5)(x + 1)$.

3. Divide $x^3 - 7x + 6$ by $x - 1$. What is the quotient?

A. $x^2 - 7x + 6$

B. $x^2 + x + 6$

C. $x^2 - x - 6$

D. $x^2 + x - 6$

D. $x^2 + x - 6$ Coefficients 1, 0, −7, 6 with c = 1: 1, 1, −6, remainder 0.

4. Use the Factor Theorem: is $x + 2$ a factor of $x^3 + 3x^2 - 4$?

A. No, since P(2) = 16

B. No, since P(−2) = −4

C. Yes, since P(−2) = 0

D. Yes, since P(2) = 0

C. Yes, since P(−2) = 0 −8 + 12 − 4 = 0, so the Factor Theorem says yes.

5. Far to the left and right, $(3x + 1)/(x^2 - 4)$ levels off at which line?

A. $y = 3$
B. $y = 1/4$
C. None
D. $y = 0$

D. $y = 0$ The top degree is smaller than the bottom degree.

6. Which vertical line does the graph of $(x - 3)/(x^2 - 9)$ never cross?

A. $x = 3$
B. $x = -3$
C. $x = 9$
D. $x = 3$ and $x = -3$

B. $x = -3$ $(x - 3)$ cancels, leaving a hole at 3 and an asymptote at -3.

CHAPTER 11

Exponential Functions

Read it in full: **Chapter 11 in the textbook**. Practice it in the lab: **Doubling Time and Half-Life** and **Compounding Toward e**.

What to know

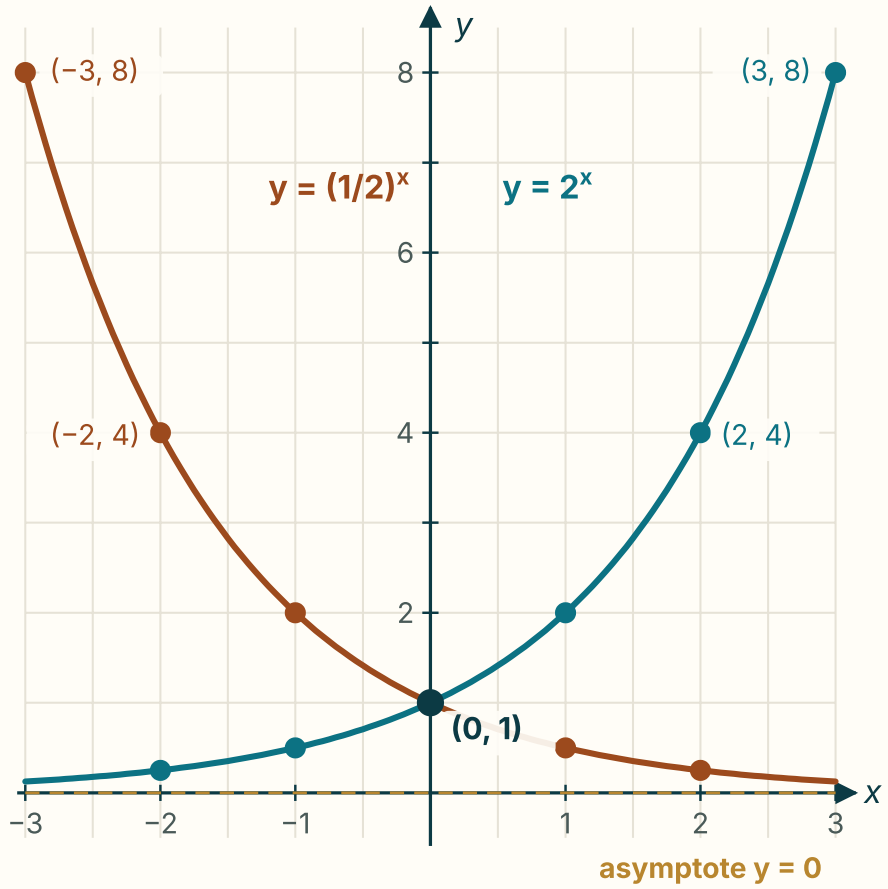
$f(x) = a \cdot b^x$ with $a \neq 0$, $b > 0$, $b \neq 1$; each step right multiplies by b. $b > 1$ grows and $0 < b < 1$ decays. The parent passes through $(0, 1)$ and $(1, b)$ with range $(0, \infty)$ and asymptote $y = 0$; in $a \cdot b^{x - h} + k$ only k moves the asymptote.

$e \approx 2.71828$ is the limit of $(1 + 1/n)^n$; continuous change is A_0e^{kt}. Percent change uses base $1 + r$; doubling uses $2^{t/d}$ and half-life uses $(1/2)^{t/h}$.

Compound interest is $P(1 + r/n)^{nt}$, continuous is Pe^{rt}, and the effective yield is the one-year factor minus 1. Percents never add across periods; round only at the end.

LOCK THESE IN

- Variable in the exponent.
- Only k moves the asymptote.
- Base is 1 + r.
- Half-life counts can be fractional.
- Continuous is the ceiling.
- Do not round early.



$f(x) = a \cdot b^x$, $b > 0$, $b \neq 1$

$b > 1$: growth

$0 < b < 1$: decay

each step right

multiplies by b

both pass through (0, 1)

domain $(-\infty, \infty)$

range $(0, \infty)$

no x-intercept

$(1/2)^x = 2^{-x}$:

a mirror image across

the y-axis

The base decides growth or decay. Each step to the right multiplies the output by b. Both curves pass through (0, 1), stay above y = 0, and are mirror images of each other across the y-axis.

Chapter 11 models

Item	Formula or rule	When to use it
Percent	$A_0(1 + r)^t$	Rate per period
Half-life	$A_0(1/2)^{t/h}$	Known half-life
Compound	$P(1 + r/n)^{nt}$	n per year
Continuous	Pe^{rt}	Continuous compounding

Worked examples

EXAMPLE 11.1 · DOUBLING WITH A FRACTIONAL COUNT

250 bacteria double every 4 hours. How many after 10 hours?

STEP 1 · MODEL

$A = 250 \cdot 2^{t/4}$.

STEP 2 · EXPONENT

10/4 = 2.5 doublings.

STEP 3 · EVALUATE

$250 \cdot 2^{2.5} \approx 250 \times 5.65685 \approx$ **1,414**, between 1,000 (8 h) and 2,000 (12 h).

EXAMPLE 11.2 · MONTHLY COMPOUNDING AND YIELD

$6,000 at 5% compounded monthly for 4 years.

STEP 1 · PIECES

r/n = 0.05/12; nt = 48.

STEP 2 · BALANCE

$6000(1 + 0.05/12)^{48} \approx 6000 \times 1.220895 \approx$ **$7,325.37**.

STEP 3 · EFFECTIVE YIELD

$(1 + 0.05/12)^{12} - 1 \approx 0.05116$, about **5.12%**.

TRAPS PEOPLE FALL INTO

- **Using the rate as the base.** 5% growth is base 1.05.
- **Moving the asymptote horizontally.** Only k moves it.
- **Adding percents.** Each period compounds.
- **Rounding r/n early.** Keep full precision.

Chapter 11 self-test · six items

1. For $y = 2(1.5)^x$, what is y at x = 2?

A. 3

B. 9

C. 6

D. 4.5

D. 4.5 $1.5^2 = 2.25$, and $2 \times 2.25 = 4.5$.

2. Which line does $y = 3^x - 4$ approach on the far left?

A. y = 0

B. y = −4

C. y = 3

D. y = 4

B. y = −4 Only the added constant moves it.

3. Match bases to solve $8^x = 32$.

A. x = 3/5

B. x = 4

C. x = 5/3

D. x = 24

C. x = 5/3 $2^{3x} = 2^5$, so 3x = 5.

4. A $20,000 car loses 10% a year. What is it worth after 3 years?

A. $14,580
B. $14,000
C. $16,200
D. $18,000

A. $14,580 $20000(0.9)^3 = 14{,}580$. Percents do not add to 30%.

5. $500 grows continuously at 3% for 10 years. About how much results?

A. $650.00
B. $674.93
C. $515.23
D. $671.96

B. $674.93 $500e^{0.3} \approx 500 \times 1.34986 \approx 674.93$.

6. What is the effective annual yield of 12% compounded monthly?

A. About 12.36%
B. Exactly 12.00%
C. About 12.68%
D. About 1.00%

C. About 12.68% $(1.01)^{12} - 1 \approx 0.1268$.

CHAPTER 12

Logarithmic Functions

Read it in full: **Chapter 12 in the textbook**. Practice it in the lab: **Evaluate a Logarithm by Inspection** and **Change of Base**.

What to know

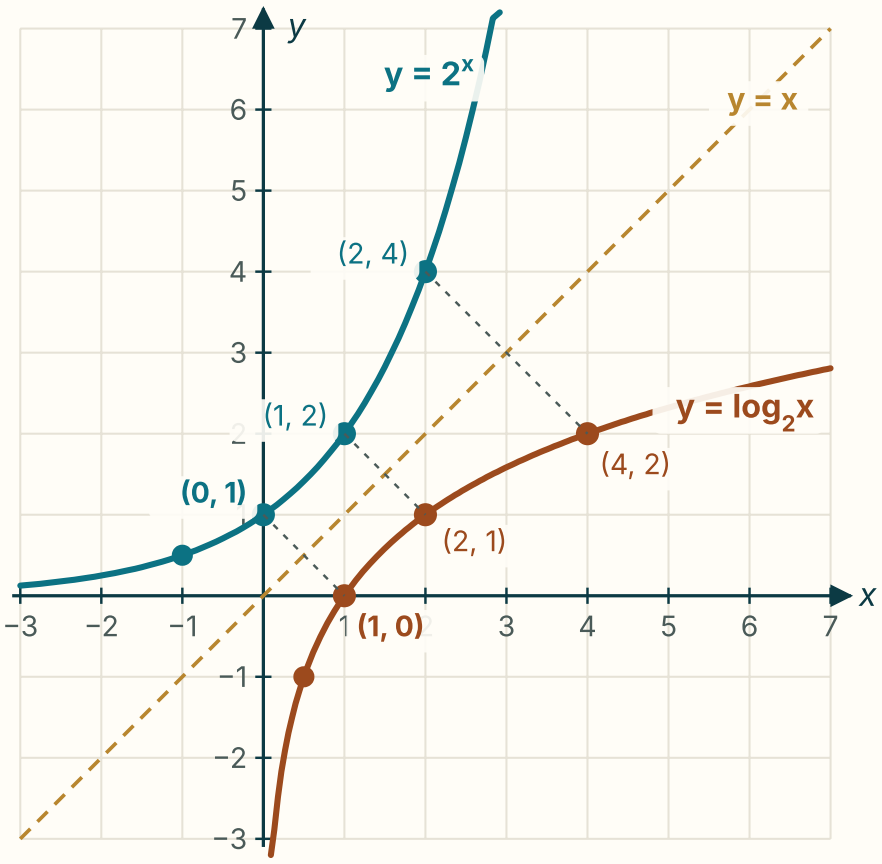
$\log_b x = y$ means $b^y = x$: a log is an exponent, and logs undo exponentials. The base is positive and not 1; the argument is positive. log is base 10 and ln is base e. $\log_b 1 = 0$, $\log_b b = 1$, $\log_b b^k = k$.

The parent log graph has domain $(0, \infty)$, range all reals, asymptote $x = 0$, and points $(1, 0)$ and $(b, 1)$; it mirrors b^x across $y = x$. Domain: argument > 0. Asymptote: argument = 0.

Product to sum, quotient to difference, power to coefficient. A sum inside a log cannot be split. Change of base: $\log_b M = \ln M / \ln b$. In log equations, condense, convert, and check for extraneous answers.

LOCK THESE IN

- A log is an exponent.
- Argument > 0.
- log is base 10; ln is base e.
- Horizontal shifts move the asymptote.
- Old base on the bottom.
- Check every candidate.



$\log_b x = y$ means $b^y = x$

a log is an exponent

swap every (x, y):

(0, 1) becomes (1, 0)

(2, 4) becomes (4, 2)

exponential:

domain all reals

range (0, ∞), HA y = 0

logarithm:

domain (0, ∞)

range all reals, VA x = 0

Inverse functions mirror across y = x. Swapping each point of $y = 2^x$ gives a point of $y = \log_2 x$. Domain and range trade places, and the horizontal asymptote y = 0 becomes the vertical asymptote x = 0.

Chapter 12 rules

Item	Formula or rule	When to use it
Definition	$\log_b x = y \Leftrightarrow b^y = x$	Converting
Product and quotient	$\log MN = \log M + \log N$; $\log(M/N) = \log M - \log N$	Expanding, condensing
Power	$\log M^p = p \log M$	Exponents out front
Change of base	$\ln M / \ln b$	Any base

Worked examples

EXAMPLE 12.1 · EVALUATING

Evaluate $\log_9 27$ and $\log_5(1/125)$.

STEP 1 · COMMON BASE

$9^y = 27$ gives $3^{2y} = 3^3$, so **3/2**.

STEP 2 · RECIPROCAL

$5^{-3} = 1/125$, so **−3**.

EXAMPLE 12.2 · AN EQUATION

Solve $\log_6 x + \log_6(x - 5) = 2$.

STEP 1 · DOMAIN

$x > 5$.

STEP 2 · CONDENSE AND CONVERT

$x(x - 5) = 36$, so $(x - 9)(x + 4) = 0$.

STEP 3 · SCREEN

−4 is extraneous; **x = 9**, since $\log_6 9 + \log_6 4 = \log_6 36 = 2$.

TRAPS PEOPLE FALL INTO

- **Pressing LN for log.** Different bases.
- **Asymptote always x = 0.** Set the argument to 0.
- **Splitting log(x + 5).** It cannot be split.
- **Keeping −4.** Its arguments are negative.

Chapter 12 self-test · six items

1. 2 to what power gives 64? That is, find $\log_2 64$.

A. 8

B. 6

C. 32

D. 5

B. 6 $2^6 = 64$.

2. Evaluate $\log(10^{-4})$.

A. 4

B. 0.0001

C. −4

D. −0.4

C. −4 log undoes the base-10 power.

3. Which inputs are allowed in $\ln(x + 7)$?

A. $(-\infty, -7)$

B. $[-7, \infty)$

C. $(7, \infty)$

D. $(-7, \infty)$

D. $(-7, \infty)$ $x + 7 > 0$ gives $x > -7$.

4. Expand $\ln[x^3/(y\sqrt{z})]$.

A. $3 \ln x - \ln y - \frac{1}{2} \ln z$

B. $3 \ln x + \ln y - \frac{1}{2} \ln z$

C. $\ln 3x - \ln y - \frac{1}{2} \ln z$

D. $3 \ln x - \ln y + \frac{1}{2} \ln z$

A. $3 \ln x - \ln y - \frac{1}{2} \ln z$ The whole denominator is subtracted, and $\sqrt{z} = z^{1/2}$.

5. Evaluate $\log_7 50$ to four decimals.

A. 1.6990
B. 2.0104
C. 0.4974
D. 7.1429

B. 2.0104 ln 50/ln 7 ≈ 3.912023/1.945910 ≈ 2.0104.

6. Solve 2 log x = log 49.

A. x = 24.5
B. x = −7 only
C. x = ±7
D. x = 7 only

D. x = 7 only $x^2 = 49$, but log(−7) is undefined.

CHAPTER 13

Exponential and Logarithmic Equations

Read it in full: **Chapter 13 in the textbook**. Practice it in the lab: **Condense, Convert, Solve** and **Growth and Decay: Solve for the Unknown**.

What to know

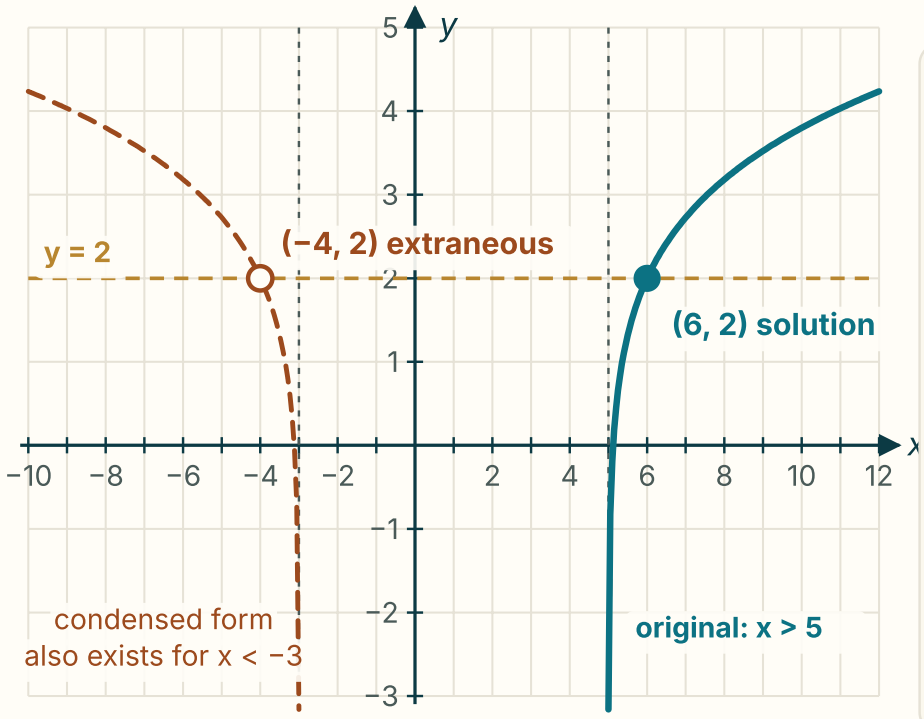
For an exponential equation, match bases if you can; otherwise isolate the exponential and take a log of both sides. b^x is never zero or negative.

For a log equation, write the domain, condense to one log, convert to exponential form, solve, and test every argument. Condensing widens the domain, which is where extraneous solutions come from.

Doubling time is ln 2/r, half-life models use $(1/2)^{t/h}$, and interest time is found by logs, then converted from periods to years. Fit $y = ab^x$ from two points: $b - 1$ is the percent change per step.

LOCK THESE IN

- Match bases first.
- Isolate before taking a log.
- Domain before solving.
- Test arguments, not answers.
- Doubling time ln 2/r.
- Constant ratios mean exponential.



$\log_3(x - 5) + \log_3(x + 3) = 2$

domain: x > 5

condense:

$(x - 5)(x + 3) = 9$

$x^2 - 2x - 24 = 0$

$x = 6$ or $x = -4$

x = 6: 0 + 2 = 2

x = −4: log of −9

$(-9)(-1) = 9$ fools

the condensed form

Condensing widens the domain. The condensed log accepts $x < -3$ because two negatives multiply to a positive. That extra branch is where the false answer $x = -4$ comes from, so check every candidate in the original.

Chapter 13 tools

Item	Formula or rule	When to use it
One-to-one	$b^u = b^v \Rightarrow u = v$	Matching bases
Take a log	$x = \ln c / \ln b$	Mismatched bases
Convert	$\log_b M = c \Rightarrow b^c = M$	One log left
Doubling	$\ln 2 / r$	Continuous growth

Worked examples

EXAMPLE 13.1 · TWO EXPONENTIAL EQUATIONS

Solve $2 \cdot 3^x + 1 = 55$, then $7^x = 20$.

STEP 1 · ISOLATE AND MATCH

$3^x = 27 = 3^3$, so **$x = 3$**.

STEP 2 · TAKE LN

$x = \ln 20 / \ln 7 \approx 2.995732 / 1.945910 \approx$ **1.5395**.

EXAMPLE 13.2 · A LOG EQUATION

Solve $\ln x + \ln(x - 2) = \ln 8$.

STEP 1 · DOMAIN

$x > 2$.

STEP 2 · CONDENSE AND MATCH

$x(x - 2) = 8$, so $(x - 4)(x + 2) = 0$.

STEP 3 · SCREEN

Keep **$x = 4$**; -2 is extraneous.

TRAPS PEOPLE FALL INTO

- **Logging before isolating.** Undo the coefficient and constant first.
- **Solving $2^x = -8$.** No solution.

- **Testing the answer instead of the arguments.** Every argument must be positive.
- **Periods reported as years.** Divide by n.

Chapter 13 self-test · six items

1. Match bases to solve $3^{x+1} = 81$.

A. x = 27

B. x = 26

C. x = 3

D. x = 4

C. x = 3 $81 = 3^4$, so $x + 1 = 4$.

2. Take ln of both sides to solve $e^{3x} = 20$.

A. x ≈ 0.9986

B. x ≈ 2.9957

C. x ≈ 6.6667

D. x ≈ 1.0986

A. x ≈ 0.9986 $3x = \ln 20 \approx 2.995732$.

3. Condense and solve $\log_2 x + \log_2(x - 6) = 4$.

A. x = 8 and x = −2

B. x = 8 only

C. x = 22

D. x = −2 only

B. x = 8 only $x(x - 6) = 16$ gives $(x - 8)(x + 2) = 0$; −2 is extraneous.

4. At 4% continuous growth, about how long until doubling?

A. 4.00 years
B. 25.00 years
C. 18.00 years
D. 17.33 years

D. 17.33 years ln 2/0.04 ≈ 17.33.

5. A half-life is 12 years. When does one eighth remain?

A. After 24 years
B. After 96 years
C. After 36 years
D. After 1.5 years

C. After 36 years $1/8 = (1/2)^3$, so 3 half-lives.

6. A model passes through (0, 50) and (2, 72). What is its growth per step?

A. 22%
B. 44%
C. 11%
D. 20%

D. 20% $b^2 = 72/50 = 1.44$, so $b = 1.2$.

CHAPTER 14

Sequences, Series and the Binomial Theorem

Read it in full: **Chapter 14 in the textbook**. Practice it in the lab: **Geometric Sequences** and **When Does an Infinite Sum Converge?**.

What to know

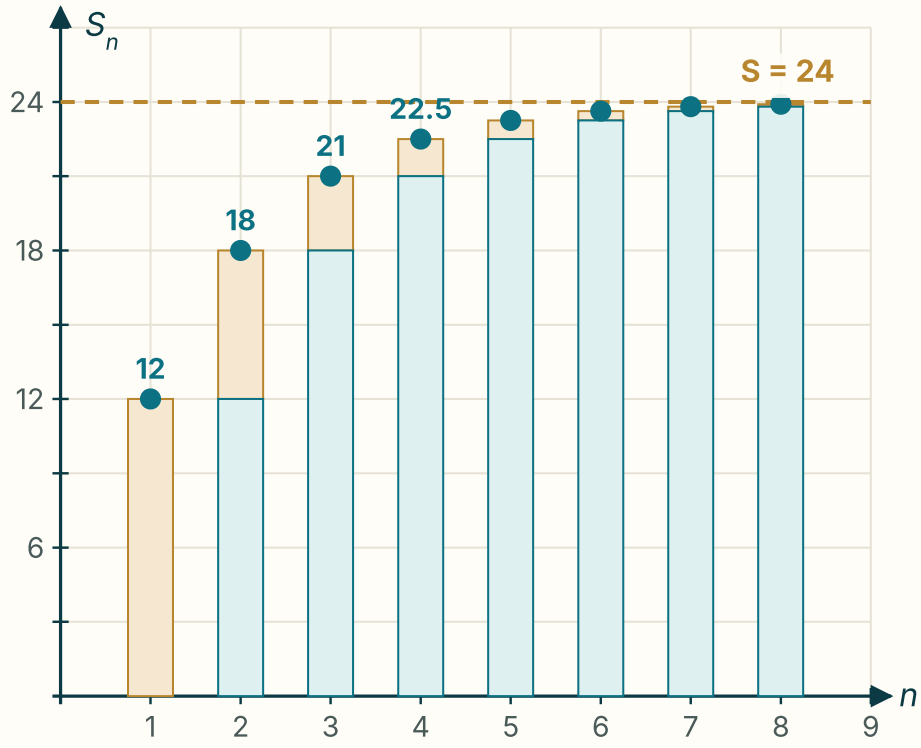
A sequence lists terms a_n by a positive integer index. Explicit formulas jump to any term; recursive formulas need a seed. Arithmetic sequences add d: $a_n = a_1 + (n - 1)d$ and $S_n = (n/2)(a_1 + a_n)$. A non-integer n means a number is not a term.

Geometric sequences multiply by r: $a_n = a_1 r^{n-1}$ and $S_n = a_1(1 - r^n)/(1 - r)$, $r \neq 1$. An infinite geometric series converges only for $|r| < 1$, to $a_1/(1 - r)$. Repeating decimals are such series.

Pascal's triangle gives $C(n, k) = n!/[k!(n - k)!]$. $(a + b)^n = \Sigma\ C(n, k)a^{n-k}b^k$: n + 1 terms with exponents adding to n. Check with x = 1.

LOCK THESE IN

- Subscript means position.
- $n - 1$, not n, in a_n.
- Reject fractional n.
- $|r| < 1$ to converge.
- $C(n, k) = n!/[k!(n - k)!]$.
- Exponents add to n.



12 + 6 + 3 + 1.5 + ...

$a_1 = 12$, $r = 1/2$

$|r| < 1$: converges

$S = a_1/(1 - r)$

$= 12/(1/2) = 24$

each new term (gold)

is half the last, so

the gap to 24 halves

If $|r| \geq 1$, the sums

never settle: diverges.

An endless sum can still have a total. With $|r| < 1$ each new term is smaller, and the running total closes half the remaining gap each time. The partial sums settle on $a_1/(1 - r) = 24$.

Chapter 14 formulas

Item	Formula or rule	When to use it
Arithmetic	$a_1 + (n - 1)d$; $(n/2)(a_1 + a_n)$	Constant difference
Geometric	a_1r^{n-1}; $a_1(1 - r^n)/(1 - r)$	Constant ratio
Infinite	$a_1/(1 - r)$	$\|r\| < 1$
Binomial term	$C(n, k)a^{n-k}b^k$	One term

Worked examples

EXAMPLE 14.1 · AN ARITHMETIC SUM

Add the first 30 terms of 3 + 8 + 13 + …

STEP 1 · LAST TERM

$a_{30} = 3 + 29(5) = 148$.

STEP 2 · SUM

$S_{30} = 15(3 + 148) =$ **2,265**.

EXAMPLE 14.2 · AN EXPANSION AND A DECIMAL

Expand $(x + 3)^4$ and write 0.7777… as a fraction.

STEP 1 · ROW 4

1, 4, 6, 4, 1: $\mathbf{x^4 + 12x^3 + 54x^2 + 108x + 81}$. At x = 1: $256 = 4^4$.

STEP 2 · SERIES

(7/10)/(1 − 1/10) = **7/9**.

TRAPS PEOPLE FALL INTO

- **Reading a_3 as a cubed.** It is the third term.
- **$a_1 + nd$.** It is $a_1 + (n - 1)d$.
- **Summing when $|r| \geq 1$.** The series diverges.
- **$(2x)^3 = 2x^3$.** It is $8x^3$.

Chapter 14 self-test · six items

1. For $a_n = (-1)^n \cdot 3n$, what is a_5?

A. −5

B. −243

C. 15

D. −15

D. −15 $(-1)^5 = -1$, and 3(5) = 15.

2. An arithmetic sequence has $a_1 = 10$ and $d = -4$. What is a_{12}?

A. −34
B. 58
C. −30
D. −38

A. −34 10 + 11(−4) = −34.

3. What is the 8th term of 3, −6, 12, …?

A. 384
B. 768
C. −384
D. −768

C. −384 r = −2, so $3(-2)^7 = 3(-128) = -384$.

4. What is the sum of 6 + 2 + 2/3 + … forever?

A. 18
B. 9
C. 8.67
D. 12

B. 9 r = 1/3, so 6/(2/3) = 9.

5. What is C(7, 2)?

A. 21
B. 42
C. 14
D. 35

A. 21 7!/(2! 5!) = (7 · 6)/2 = 21.

6. Write 0.121212… as a fraction in lowest terms.

A. 12/100
B. 4/33
C. 1/12
D. 6/55

B. 4/33 (12/100)/(99/100) = 12/99 = 4/33.

ONE PAGE

Cram sheet

Every formula in the course, sorted by topic. Cover the right column and rebuild each row. If a row will not come back, go to that chapter's exam guide.

Numbers, equations and lines

Item	Formula or rule	Note
Exponent rules	$a^m a^n = a^{m+n}$; $(a^m)^n = a^{mn}$; $a^{-n} = 1/a^n$	$a^0 = 1$, $a \neq 0$
Scientific notation	$a \times 10^n$, $1 \leq \lvert a \rvert < 10$	Match exponents to add
Order of operations	Grouping, exponents, × ÷, + −	Left to right within a level
Inequalities	Flip for × or ÷ by a negative	Bracket includes
Absolute value	$\lvert u \rvert < c$: and; $\lvert u \rvert > c$: or	Isolate first
Slope	$(y_2 - y_1)/(x_2 - x_1)$	Undefined if vertical
Line forms	$y = mx + b$; $y - y_1 = m(x - x_1)$; $Ax + By = C$	Point-slope builds
Parallel and perpendicular	Equal slopes; product −1	Flip and negate

Functions and systems

Item	Formula or rule	Note
Function	One output per input	Vertical line test
Domain	No zero denominators, no negative even radicands	Find by exclusion
Transformations	$a \cdot f(x - h) + k$	Inside reversed, outside literal
Systems	One, none or infinitely many	Compare slopes
Elimination and substitution	Cancel a variable, or swap an expression	Check every equation
Mixture	$p_1x + p_2y = p_{mix}(x + y)$	Track the substance

Polynomials, rationals and radicals

Item	Formula or rule	Note
Special products	$(a \pm b)^2 = a^2 \pm 2ab + b^2$; $(a + b)(a - b) = a^2 - b^2$	Never $a^2 + b^2$
Cubes	$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$	SOAP
AC method	Product ac, sum b	GCF first
Rational expressions	Factor, cancel factors, keep restrictions	LCD: highest powers
Rational equations	Clear with the LCD, screen candidates	Extraneous if excluded
Radicals	$\sqrt{(ab)} = \sqrt{a}\sqrt{b}$; $a^{m/n} = (\sqrt[n]{a})^m$	Conjugate for two terms
Radical equations	Isolate, raise, solve, check	Squaring invents answers

Quadratic, polynomial and rational functions

Item	Formula or rule	Note
Quadratic formula	$x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$	Standard form first
Discriminant	$b^2 - 4ac$	+ two, 0 one, − none
Vertex	$x = -b/(2a)$; $a(x - h)^2 + k$	Halfway between roots
Projectile	$-16t^2 + v_0t + h_0$	Feet, seconds
End behavior	Even agrees, odd disagrees, negative flips	Leading term rules
Multiplicity	Odd crosses, even touches	Sum = degree
Remainder and Factor	Remainder P(c); 0 means factor	Synthetic division
Rational graph	Hole if canceled, VA if not; HA by degrees	Domain before canceling

Exponentials and logarithms

Item	Formula or rule	Note
Exponential	$a \cdot b^x$; asymptote $y = k$ in $a \cdot b^{x-h} + k$	$b > 1$ grows
Growth and decay	$A_0(1 + r)^t$; $A_0 2^{t/d}$; $A_0(1/2)^{t/h}$	Percents compound
Interest	$P(1 + r/n)^{nt}$; Pe^{rt}	Yield = factor − 1
Logarithm	$\log_b x = y \Leftrightarrow b^y = x$	Argument > 0
Log properties	Product to sum; quotient to difference; power to coefficient	No splitting sums
Change of base	$\log_b M = \ln M / \ln b$	Old base on the bottom
Equations	Match bases or take logs; condense, convert, check	Extraneous from condensing
Doubling time	$\ln 2 / r$	Independent of A_0

Sequences and series

Item	Formula or rule	Note
Arithmetic	$a_1 + (n - 1)d$; $S_n = (n/2)(a_1 + a_n)$	n a positive integer
Geometric	$a_1 r^{n-1}$; $S_n = a_1(1 - r^n)/(1 - r)$	$r \neq 1$
Infinite geometric	$a_1/(1 - r)$	$\lvert r \rvert < 1$ only
Binomial	$C(n, k) = n!/[k!(n - k)!]$; term $C(n, k)a^{n-k}b^k$	$n + 1$ terms

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