COLLEGE ALGEBRA · STUDY GUIDE

Exam 7 Study Guide

This guide covers chapters 13 and 14. You will solve exponential equations two ways, solve logarithmic equations while guarding the domain, and use both on doubling, half-life, interest and model-fitting problems. Then you will write sequences explicitly and recursively, add arithmetic and geometric series, decide when an endless sum has a total, and expand powers with the binomial theorem. Work one chapter at a time. Read the "what to know" paragraphs, copy the tables out by hand, then try each worked example with the steps covered. Finish each chapter with its ten self-test items. The cram sheet at the end puts every formula in one place.

WHAT THIS GUIDE COVERS

- Chapter 13 · matching bases, taking logs, condensing and converting log equations, domains and extraneous solutions, doubling time, half-life, compound interest and fitting models.
- Chapter 14 · sequence notation, explicit and recursive formulas, arithmetic and geometric sequences and series, infinite geometric series and repeating decimals, Pascal's triangle and the binomial theorem.

CHAPTER 13

Exponential and Logarithmic Equations

Read it in full: **Chapter 13 in the textbook**. Practice it in the lab: **Same Base, Same Exponent** and **Check the Domain**.

What to know

An **exponential equation** has the variable in an exponent. There are two doors. Door 1: if both sides can be written as powers of one base, do it and set the exponents equal, using the **one-to-one property** $b^u = b^v \Rightarrow u = v$. Try this door first. Since b^x is always positive, an equation like $3^x = -7$ has no solution at all.

Door 2: **isolate the exponential first**, then take a log of both sides and bring the exponent down with $\log(M^p) = p \log M$. For $4 \cdot 5^x + 3 = 87$, subtract 3 and divide by 4 to get $5^x = 21$, then $x = \ln 21/\ln 5 \approx 1.8917$. Any log base gives the same answer; use ln when the base is e.

A **logarithmic equation** has the variable inside a log. Get down to one logarithm, then convert: $\log_b M = c$ means $b^c = M$ ("the base raised to the answer gives the inside"). With two or more logs, condense first using the product, quotient and power rules, which need one shared base.

Write the **domain** before you solve: every argument must be positive. If the variable has a negative coefficient, dividing the domain inequality by a negative flips it: $4 - x > 0$ gives $x < 4$. If condensing creates a fraction, name its excluded value too.

An **extraneous solution** satisfies an equation you made along the way but not the original. It comes from a legal step: condensing $\log M + \log N$ into $\log(MN)$ only requires the product to be positive, and two negatives multiply to a positive. The power rule does the same: $\log(x^2)$ accepts negatives that $2 \log x$ does not. Test each candidate by checking that every original argument is positive. A positive answer is not the test; a positive argument is.

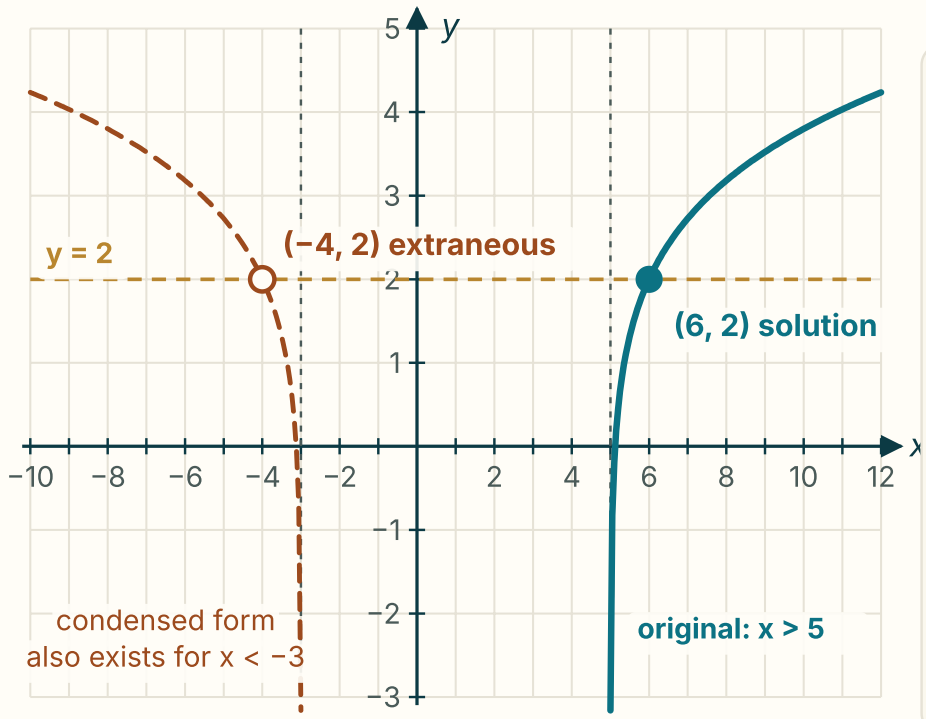
Rejecting a candidate is part of doing the problem right. Exponential equations do not create extraneous answers this way, because b^x accepts every real x; the domain chore belongs to logarithms.

Applications are the same equations in costume. Continuous growth $A = A_0e^{rt}$ gives a **doubling time** of $\ln 2/r$, whatever the starting amount. A known half-life uses $A = A_0(1/2)^{t/h}$, and the ratio A/A_0 is all you need. Compound interest $A = P(1 + r/n)^{nt}$ gives an answer in periods first; convert to years and think about when interest actually posts.

To model, fit $y = ab^x$ from two points: a point at $x = 0$ gives a, and the other gives b. a is the initial value, b is the growth factor, and $b - 1$ is the percent change per step. In $y = a + b \ln x$, every doubling of x adds the same $b \ln 2$: diminishing returns. Taking ln of $y = ab^x$ gives a straight line, $\ln y = \ln a + x \ln b$. Constant differences in data mean linear; constant ratios mean exponential.

LOCK THESE IN

- Door 1: match bases, set exponents equal.
- Door 2: isolate the exponential, then take a log.
- b^x = negative or zero: no solution.
- Log equations: condense to one log, then convert.
- Write the domain first; arguments must be positive.
- Condensing widens the domain, so test every candidate.
- Doubling time ln 2/r; half-life model $A_0(1/2)^{t/h}$.
- In $y = ab^x$, $b - 1$ is the percent change per step.



$\log_3(x - 5) + \log_3(x + 3) = 2$

domain: x > 5

condense:

$(x - 5)(x + 3) = 9$

$x^2 - 2x - 24 = 0$

$x = 6$ or $x = -4$

x = 6: 0 + 2 = 2

x = −4: log of −9

$(-9)(-1) = 9$ fools

the condensed form

Condensing widens the domain. The condensed log accepts $x < -3$ because two negatives multiply to a positive. That extra branch is where the false answer $x = -4$ comes from, so check every candidate in the original.

Solving tools and when to use them

Tool	Formula or move	When to use it
One-to-one property	$b^u = b^v \Rightarrow u = v$	Both sides are powers of one base
Take a log	$b^x = c \Rightarrow x = \ln c / \ln b$	Bases do not match; isolate first
Convert	$\log_b M = c \Rightarrow b^c = M$	One log equals a number
Condense	Product, quotient, power rules	Two or more logs on one side
Equal logs	$\log_b M = \log_b N \Rightarrow M = N$	One log on each side
Domain	Every argument > 0	Before solving any log equation
Doubling time	$t = \ln 2 / r$	Continuous growth at rate r
Half-life	$A = A_0(1/2)^{t/h}$	Decay with known half-life
Interest time	$t = \ln(A/P)/[n \ln(1 + r/n)]$	Time to reach a balance
Two-point fit	$a = y(0)$; $b = (y_2/a)^{1/x_2}$	Building $y = ab^x$

Candidates and verdicts

Equation	Domain	Candidates	Verdict
$\log_2(x + 1) + \log_2(x - 3) = 5$	$x > 3$	7, −5	Keep 7
$\ln(x + 8) - \ln(x - 1) = \ln 4$	$x > 1$	4	Keep 4
$2 \log x = \log 36$	$x > 0$	6, −6	Keep 6
$3 \cdot 4^x - 5 = 40$	All reals	1.9535	No domain issue

Worked examples

WORKED EXAMPLE 1 · MATCHING BASES

Solve $9^{x+1} = 27^{x-2}$.

STEP 1 · COMMON BASE

$9 = 3^2$ and $27 = 3^3$, so $3^{2x+2} = 3^{3x-6}$.

STEP 2 · SET EXPONENTS EQUAL

$2x + 2 = 3x - 6$, so $\mathbf{x = 8}$.

CHECK

$9^9 = 3^{18}$ and $27^6 = 3^{18}$.

WORKED EXAMPLE 2 · ISOLATE, THEN TAKE A LOG

Solve $3 \cdot 4^x - 5 = 40$.

STEP 1 · ISOLATE

Add 5: $3 \cdot 4^x = 45$. Divide by 3: $4^x = 15$.

STEP 2 · TAKE LN

$x \ln 4 = \ln 15$.

STEP 3 · DIVIDE

$x = 2.708050/1.386294 \approx \mathbf{1.9535}$.

REASONABLE?

$4^1 = 4$ and $4^2 = 16$, so x is just under 2.

WORKED EXAMPLE 3 · A LOG EQUATION WITH AN EXTRANEOUS CANDIDATE

Solve $\log_2(x + 1) + \log_2(x - 3) = 5$.

STEP 1 · DOMAIN

$x + 1 > 0$ and $x - 3 > 0$, so $x > 3$.

STEP 2 · CONDENSE AND CONVERT

$(x + 1)(x - 3) = 2^5 = 32$.

STEP 3 · SOLVE

$x^2 - 2x - 35 = 0 = (x - 7)(x + 5)$, so $x = 7$ or $x = -5$.

STEP 4 · SCREEN

-5 makes both arguments negative: extraneous. $\mathbf{x = 7}$: $\log_2 8 + \log_2 4 = 3 + 2 = 5$.

WORKED EXAMPLE 4 · A QUOTIENT OF LOGS

Solve $\ln(x + 8) - \ln(x - 1) = \ln 4$.

STEP 1 · DOMAIN

$x > 1$. The fraction $(x + 8)/(x - 1)$ excludes $x = 1$, already barred.

STEP 2 · CONDENSE AND MATCH

$\ln[(x + 8)/(x - 1)] = \ln 4$, so $(x + 8)/(x - 1) = 4$.

STEP 3 · SOLVE

$x + 8 = 4x - 4$, so $12 = 3x$ and **$x = 4$**, which clears $x > 1$.

CHECK

$\ln 12 - \ln 3 = \ln(12/3) = \ln 4$.

WORKED EXAMPLE 5 · DOUBLING TIME AND RADIOCARBON AGE

Find the doubling time at a continuous 5% rate. Then date a sample that keeps 30% of its carbon-14 (half-life 5,730 years).

STEP 1 · DOUBLING TIME

$t = \ln 2/0.05 = 0.693147/0.05 \approx$ **13.86 years**.

STEP 2 · SET UP THE DECAY

$0.30 = (1/2)^{t/5730}$. The starting amount cancels.

STEP 3 · TAKE LN

$t/5730 = \ln 0.30/\ln 0.5 = (-1.203973)/(-0.693147) = 1.736966$.

STEP 4 · SOLVE

$t = 5730 \times 1.736966 \approx$ **9,953 years**.

REASONABLE?

30% is between one quarter (2 half-lives, 11,460 years) and one half (5,730 years).

WORKED EXAMPLE 6 · TIME TO REACH A BALANCE

$2,500 earns 6% compounded quarterly. When does it reach $4,000?

STEP 1 · SET UP

$4000 = 2500(1.015)^{4t}$, so $(1.015)^{4t} = 1.6$.

STEP 2 · TAKE LN

$4t \cdot \ln 1.015 = \ln 1.6$, so $4t = 0.470004/0.0148886 \approx 31.568$ quarters.

STEP 3 · CONVERT

t ≈ **7.89 years**. Interest posts quarterly, so the balance first passes $4,000 at the 32nd posting, 8 years.

TRAPS PEOPLE FALL INTO

- **Taking a log before isolating the exponential.** Undo the coefficient and constant first.
- **Thinking the "wrong" log was used.** ln and log give the same answer.
- **Trying to solve $2^x = -8$.** b^x is always positive; there is no solution.
- **Converting before condensing.** Two logs must become one first.
- **Writing the domain at the end, or not at all.** Write it first, then screen.
- **Testing whether the answer is positive.** Test whether every argument is positive.
- **Skipping the flip in a domain inequality.** $4 - x > 0$ gives $x < 4$.
- **Reporting periods as years.** Divide by n, and think about when interest posts.

Chapter 13 self-test · ten items

1. Match bases to solve $2^{3x} = 16^{x-1}$.

A. $x = 4/7$

B. $x = 1$

C. $x = -4$

D. $x = 4$

D. $x = 4$ $16 = 2^4$, so $3x = 4x - 4$ and $x = 4$.

2. Solve $5^x = 40$ to four decimal places.

A. $x \approx 8.0000$

B. $x \approx 0.4363$

C. $x \approx 2.2920$

D. $x \approx 1.6021$

C. $x \approx 2.2920$ $x = \ln 40/\ln 5 \approx 3.688879/1.609438 \approx 2.2920$. It sits between 2 and 3, since $25 < 40 < 125$.

3. Solve $3e^{2x} = 30$.

A. $x \approx 2.3026$

B. $x \approx 1.1513$

C. $x \approx 5.0000$

D. $x \approx 1.7006$

B. $x \approx 1.1513$ Isolate: $e^{2x} = 10$. Then $2x = \ln 10 \approx 2.302585$, so $x \approx 1.1513$.

4. Count the real solutions of $2^x = -8$.

A. None at all

B. One: $x = 3$

C. Two: $x = \pm 3$

D. One: $x = -3$

A. None at all A positive base raised to any real power is positive, so it can never equal -8.

5. Convert $\log_4(3x - 2) = 2$ to exponential form and solve.

A. $x = 4/3$

B. $x = 18$

C. $x = 6$

D. $x = 10/3$

C. $x = 6$ Convert: $4^2 = 3x - 2$, so $16 = 3x - 2$ and $x = 6$. The argument 16 is positive.

6. Solve log(x) + log(x − 15) = 2.

A. x = 20 only
B. No real value
C. x = −5 only
D. x = 20 and x = −5

A. x = 20 only x(x − 15) = 100 gives (x − 20)(x + 5) = 0. The domain is x > 15, so −5 is extraneous.

7. Which inequality describes the inputs allowed in log(6 − 2x)?

A. x > 3
B. x < 3
C. x < 6
D. x > −3

B. x < 3 6 − 2x > 0 gives −2x > −6. Dividing by −2 flips it: x < 3.

8. Money grows continuously at 7% a year. About how long does it take to double?

A. 14.29 years
B. 7.00 years
C. 10.24 years
D. 9.90 years

D. 9.90 years Doubling time = ln 2/r = 0.693147/0.07 ≈ 9.90 years, whatever the starting amount.

9. A substance has a half-life of 10 days. When does 25% of it remain?

A. After 7.5 days
B. After 20 days
C. After 25 days
D. After 40 days

B. After 20 days 25% is one quarter, which is $(1/2)^2$, so two half-lives: 20 days.

10. In the model $y = 300(0.92)^t$, what does 0.92 tell you?

A. An 8% loss per step
B. An 8% gain per step
C. A 92% loss per step
D. A 92% gain per step

A. An 8% loss per step $b < 1$ is a decay factor, and $1 - b = 0.08$ is the fraction lost each step.

CHAPTER 14

Sequences, Series and the Binomial Theorem

Read it in full: **Chapter 14 in the textbook**. Practice it in the lab: **Arithmetic Sequences** and **Pascal's Triangle and the Binomial Theorem**.

What to know

A **sequence** is an ordered list, a function whose inputs are the positive integers. Terms are written a_1, a_2, …, a_n, where n is the index. The subscript is a position, not an exponent. An **explicit formula** gives a_n straight from n, so any term takes one substitution. Watch for excluded values such as n = 4 in $10/(n - 4)$, and use $(-1)^n$ to alternate signs.

A **recursive formula** builds each term from earlier ones and always needs a seed. $a_1 = 4$ and $a_n = 2a_{n-1} + 1$ gives 4, 9, 19, 39, 79. The Fibonacci sequence needs two seeds because its rule reaches

back two steps. The same list can have both kinds of formula.

An **arithmetic sequence** adds a common difference d each step; test by subtracting consecutive terms. $a_n = a_1 + (n - 1)d$. You add d one fewer time than the term number, and plugging in n = 1 must return a_1. Arithmetic sequences graph as points on a straight line with slope d.

A **series** is a sum of terms. In sigma notation, $\Sigma_{k=1}^{n} a_k$ adds a_k for k from 1 to n. The arithmetic sum is $S_n = (n/2)(a_1 + a_n)$, the average of the ends times the number of terms, or $(n/2)[2a_1 + (n - 1)d]$. To ask whether a number is a term, solve for n and reject any answer that is not a positive whole number.

A **geometric sequence** multiplies by a common ratio r each step; test by dividing consecutive terms. $a_n = a_1 r^{n-1}$, with $r \neq 0$. A negative r alternates the signs. The finite sum is $S_n = a_1(1 - r^n)/(1 - r)$ for $r \neq 1$; if r = 1, $S_n = n \cdot a_1$.

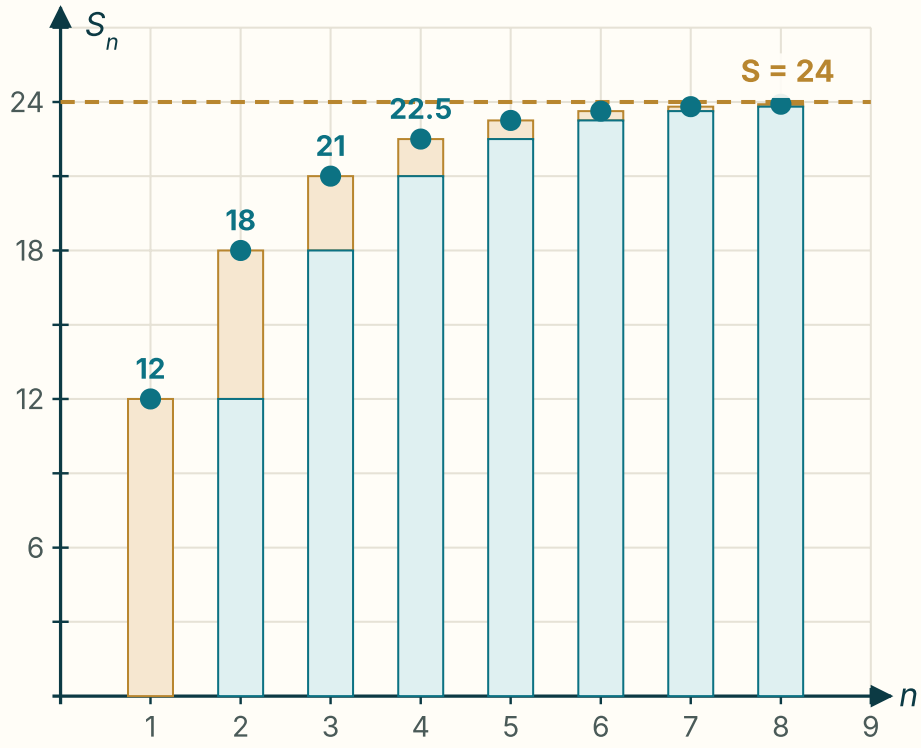
An **infinite geometric series** converges only when $|r| < 1$, that is $-1 < r < 1$, because then r^n shrinks to 0. Its sum is $S = a_1/(1 - r)$. If $|r| \geq 1$ it diverges and has no sum. Every repeating decimal is a convergent geometric series: 0.272727… = (27/100)/(1 − 1/100) = 27/99 = 3/11.

Pascal's triangle starts and ends each row with 1, and each inside entry is the sum of the two above it. Row n has n + 1 entries. The **binomial coefficient** $C(n, k) = n!/[k!(n - k)!]$, with 0! = 1, is entry k of row n, counting from 0. C(n, k) = C(n, n − k).

The **binomial theorem**: $(a + b)^n = \Sigma\, C(n, k)a^{n-k}b^k$, k from 0 to n. The power on a falls, the power on b rises, and they always add to n. For a subtraction, use b as a negative number in parentheses and the signs alternate. To find one term, solve for k first. Check any expansion by counting n + 1 terms and substituting x = 1.

LOCK THESE IN

- n is a positive integer; a subscript is a position.
- Recursive formulas need a seed.
- Arithmetic: $a_n = a_1 + (n - 1)d$; $S_n = (n/2)(a_1 + a_n)$.
- Solving for n must give a positive whole number.
- Geometric: $a_n = a_1 r^{n-1}$; $S_n = a_1(1 - r^n)/(1 - r)$.
- Infinite: $|r| < 1$ converges to $a_1/(1 - r)$.
- $C(n, k) = n!/[k!(n - k)!]$; exponents in each term add to n.
- Check expansions with x = 1.



12 + 6 + 3 + 1.5 + ...

$a_1 = 12$, $r = 1/2$

$|r| < 1$: converges

$S = a_1/(1 - r)$

$= 12/(1/2) = 24$

each new term (gold) is half the last, so the gap to 24 halves

If $|r| \geq 1$, the sums never settle: diverges.

An endless sum can still have a total. With $|r| < 1$ each new term is smaller, and the running total closes half the remaining gap each time. The partial sums settle on $a_1/(1 - r) = 24$.

Sequence and series formulas, with when to use them

Formula	Statement	When to use it
Arithmetic term	$a_n = a_1 + (n - 1)d$	Constant difference d
Arithmetic sum	$S_n = (n/2)(a_1 + a_n)$	Last term known
Arithmetic sum, alternate	$S_n = (n/2)[2a_1 + (n - 1)d]$	Last term unknown
Geometric term	$a_n = a_1 r^{n-1}$	Constant ratio r
Geometric sum	$S_n = a_1(1 - r^n)/(1 - r)$	n terms, $r \neq 1$
Infinite geometric sum	$S = a_1/(1 - r)$	Only when $\lvert r \rvert < 1$
Factorial	$n! = n(n - 1)\cdots 1$; $0! = 1$	Binomial coefficients
Binomial coefficient	$C(n, k) = n!/[k!(n - k)!]$	Row n, position k
Binomial theorem	$(a + b)^n = \Sigma\, C(n, k)a^{n-k}b^k$	Expanding a power
One term	$C(n, k)a^{n-k}b^k$	Find k from the power wanted

Pascal's triangle, rows 0 to 6

Row	Coefficients	Terms
0	1	1
1	1 1	2
2	1 2 1	3
3	1 3 3 1	4
4	1 4 6 4 1	5
5	1 5 10 10 5 1	6
6	1 6 15 20 15 6 1	7

Worked examples

WORKED EXAMPLE 1 · EXPLICIT AND RECURSIVE FORMULAS

List the first four terms of $a_n = 3n + 4$ and find a_{30}. Then list five terms of $a_1 = 2$, $a_n = 3a_{n-1} - 1$.

STEP 1 · SUBSTITUTE N = 1 TO 4

7, 10, 13, 16.

STEP 2 · JUMP AHEAD

$a_{30} = 3(30) + 4 =$ **94**.

STEP 3 · RUN THE RECURSION

2; $3(2) - 1 = 5$; $3(5) - 1 = 14$; $3(14) - 1 = 41$; $3(41) - 1 = 122$.

NOTE

The explicit formula reached a_{30} in one step; the recursion needs every term before.

WORKED EXAMPLE 2 · AN ARITHMETIC SEQUENCE AND ITS SUM

For 12, 9, 6, …, find a_{25} and S_{25}.

STEP 1 · COMMON DIFFERENCE

$d = 9 - 12 = -3$.

STEP 2 · 25TH TERM

$a_{25} = 12 + 24(-3) = 12 - 72 =$ **−60**.

STEP 3 · SUM

$S_{25} = (25/2)(12 + (-60)) = 12.5(-48) =$ **−600**.

CHECK N = 1

$12 + 0(-3) = 12$.

WORKED EXAMPLE 3 · IS IT A TERM?

Is 200 a term of 8, 14, 20, …? Is 100?

STEP 1 · SET UP FOR 200

$200 = 8 + 6(n - 1)$, so $192 = 6(n - 1)$ and $n - 1 = 32$.

STEP 2 · READ N

$n = 33$, a positive whole number, so **200 is the 33rd term**.

STEP 3 · SET UP FOR 100

$92 = 6(n - 1)$, so $n - 1 = 15.33\ldots$

STEP 4 · REJECT

There is no 16.33rd term, so **100 is not a term**.

WORKED EXAMPLE 4 · A GEOMETRIC SUM

For 2, 6, 18, …, find a_7 and S_7.

STEP 1 · RATIO

$r = 6/2 = 3$, confirmed by $18/6 = 3$.

STEP 2 · 7TH TERM

$a_7 = 2 \cdot 3^6 = 2 \cdot 729 =$ **1,458**.

STEP 3 · SUM

$S_7 = 2(1 - 3^7)/(1 - 3) = 2(1 - 2187)/(-2) = 2(-2186)/(-2) =$ **2,186**.

WORKED EXAMPLE 5 · INFINITE SERIES AND A REPEATING DECIMAL

Find the sum of $18 - 6 + 2 - \ldots$, then write $0.454545\ldots$ as a fraction.

STEP 1 · RATIO AND TEST

$r = -6/18 = -1/3$, and $|-1/3| < 1$, so it converges.

STEP 2 · SUM

$S = 18/(1 - (-1/3)) = 18/(4/3) =$ **13.5**. Subtracting a negative becomes adding.

STEP 3 · DECIMAL AS A SERIES

$0.4545\ldots = 45/100 + 45/10{,}000 + \ldots$, with $a_1 = 45/100$ and $r = 1/100$.

STEP 4 · SUM AND REDUCE

$(45/100)/(99/100) = 45/99 =$ **5/11**.

WORKED EXAMPLE 6 · A BINOMIAL EXPANSION AND ONE TERM

Expand $(x - 2)^5$. Then find the coefficient of x^3 in $(2x + 1)^6$.

STEP 1 · ROW 5

1, 5, 10, 10, 5, 1 with $a = x$ and $b = -2$.

STEP 2 · TERM BY TERM

$x^5 + 5x^4(-2) + 10x^3(4) + 10x^2(-8) + 5x(16) + (-32)$.

STEP 3 · ANSWER

$x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$. Check at x = 1: $(-1)^5 = -1$ and $1 - 10 + 40 - 80 + 80 - 32 = -1$.

STEP 4 · ONE TERM: FIND K

The power on 2x is 6 − k = 3, so k = 3.

STEP 5 · COMPUTE

$C(6, 3)(2x)^3(1)^3 = 20 \cdot 8x^3 = 160x^3$. The coefficient is **160**.

TRAPS PEOPLE FALL INTO

- **Reading a_3 as a cubed.** It is the third term.
- **Forgetting the seed.** A recursive rule with no starting value defines nothing.
- **Writing $a_{20} = a_1 + 20d$.** It is $a_1 + 19d$; check with n = 1.
- **Accepting a fractional n.** A number with n = 24.75 is not a term.
- **Using the infinite sum when $|r| \geq 1$.** The series diverges; there is no sum.
- **Using the second term as a_1.** a_1 is the very first term of the series.
- **Writing $(2x)^3$ as $2x^3$.** The power hits the 2 as well: $8x^3$.
- **Dropping the sign of b.** In $(2x - 3)^n$, use b = −3 in parentheses.

Chapter 14 self-test · ten items

1. For $a_n = 2n^2 - 1$, what is a_4?

A. 7

B. 15

C. 31

D. 63

C. 31 $2(4)^2 - 1 = 2(16) - 1 = 31$. Squaring first matters: $(2 \cdot 4)^2 - 1$ would give 63.

2. If $a_1 = 5$ and $a_n = a_{n-1} - 4$, what is a_4?

A. −11
B. −3
C. 1
D. −7

D. −7 5, 1, −3, −7. Each step subtracts 4.

3. What is the 20th term of 4, 11, 18, …?

A. 144
B. 137
C. 130
D. 140

B. 137 d = 7, so $a_{20} = 4 + 19(7) = 137$. Using 20d gives 144.

4. What is 1 + 2 + 3 + … + 60?

A. 1,830
B. 3,660
C. 3,600
D. 1,800

A. 1,830 S = (60/2)(1 + 60) = 30(61) = 1,830.

5. How many terms are in 6, 10, 14, …, 98?

A. 92

B. 25

C. 23

D. 24

D. 24 98 = 6 + 4(n − 1), so 92 = 4(n − 1), n − 1 = 23 and n = 24.

6. What is the 6th term of 81, 27, 9, …?

A. 1/9

B. 1/3

C. 3

D. 1

B. 1/3 r = 1/3, so $a_6 = 81(1/3)^5 = 81/243 = 1/3$.

7. What is the sum of the first 6 terms of 5 + 10 + 20 + …?

A. 155

B. 320

C. 315

D. 160

C. 315 $S_6 = 5(1 - 2^6)/(1 - 2) = 5(-63)/(-1) = 315$.

8. What is the sum of 20 + 15 + 11.25 + … forever?

A. 80
B. 120
C. 46.25
D. 60

A. 80 r = 15/20 = 3/4 and |r| < 1, so S = 20/(1 − 3/4) = 20/(1/4) = 80.

9. Which infinite series diverges?

A. 0.9 + 0.09 + 0.009 + …
B. 10 + 5 + 2.5 + …
C. 3 − 1 + 1/3 − …
D. 3 − 6 + 12 − 24 + …

D. 3 − 6 + 12 − 24 + … Its ratio is −2, and |−2| ≥ 1. The other ratios are −1/3, 1/2 and 1/10.

10. In the expansion of $(x + 2)^6$, which number multiplies x^4?

A. 60
B. 30
C. 240
D. 15

A. 60 The power on x is 6 − k = 4, so k = 2: C(6, 2) · 2^2 = 15 · 4 = 60.

ONE PAGE

Cram sheet

Everything from chapters 13 and 14 in one place. Cover the right column and see if you can rebuild each row.

Equations

Item	Rule	Note
Match bases	$b^u = b^v \Rightarrow u = v$	Try first
Take a log	$x = \ln c / \ln b$	Isolate the exponential first
Convert	$\log_b M = c \Rightarrow b^c = M$	After condensing to one log
Domain	Arguments > 0	Write it first; flip if dividing by a negative
Extraneous	Condensing widens the domain	Test arguments in the original
No solution	$b^x = 0$ or negative	$b^x > 0$ always

Formulas

Name	Formula	When to use it
Doubling time	$\ln 2 / r$	Continuous growth
Half-life	$A_0(1/2)^{t/h}$	Known half-life
Interest time	$\ln(A/P) / [n \ln(1 + r/n)]$	Periods compounded
Arithmetic	$a_1 + (n - 1)d$; $(n/2)(a_1 + a_n)$	Constant difference
Geometric	$a_1 r^{n-1}$; $a_1(1 - r^n)/(1 - r)$	Constant ratio, $r \neq 1$
Infinite geometric	$a_1/(1 - r)$	$\lvert r \rvert < 1$ only
Binomial coefficient	$n!/[k!(n - k)!]$	Row n of Pascal
General term	$C(n, k)a^{n-k}b^k$	One term of $(a + b)^n$

Checks

Situation	The check
Log equation	Every original argument positive for each candidate.
Term question	n must be a positive whole number.
Infinite series	Find r and confirm $-1 < r < 1$ before using the formula.
Binomial expansion	n + 1 terms, exponents add to n, and x = 1 matches.

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