COLLEGE ALGEBRA · STUDY GUIDE

Exam 6 Study Guide

This guide covers chapters 11 and 12. You will read exponential functions, graph growth and decay, meet the number e and build doubling, half-life and compound interest models. Then you will treat a logarithm as an exponent, evaluate and graph logs, use the three log properties and change of base, and solve log equations while checking for extraneous answers. Work one chapter at a time. Read the "what to know" paragraphs, copy the tables out by hand, then try each worked example with the steps covered. Finish each chapter with its ten self-test items. The cram sheet at the end puts every formula in one place.

WHAT THIS GUIDE COVERS

- Chapter 11 · the exponential function and its base rules, graphs and transformations, the number e, growth and decay models, doubling time and half-life, and compound interest.
- Chapter 12 · the logarithm as an exponent, converting forms, common and natural logs, evaluating by inspection, log graphs and asymptotes, the three properties, expanding, condensing, change of base and log equations.

CHAPTER 11

Exponential Functions

Read it in full: **Chapter 11 in the textbook**. Practice it in the lab: **The Base Decides Everything** and **Compound Interest, Four Ways**.

What to know

An **exponential function** has the form $f(x) = a \cdot b^x$ with $a \neq 0$, $b > 0$ and $b \neq 1$. The variable sits in the exponent. That is the difference from a power function such as x^2, where the variable is the base. Ask "upstairs or downstairs?" first. a is the **initial value**, because $f(0) = a \cdot b^0 = a$.

A linear function adds the same amount each step; an exponential multiplies by the same factor b each step. The base rules exist for reasons: a negative base makes fractional inputs undefined, $b = 1$ gives a constant, and $a = 0$ gives a flat line. $b > 1$ is **growth** and $0 < b < 1$ is **decay**. A negative exponent gives a small positive value, never a negative one.

To build $f(x) = a \cdot b^x$ from data, use the point with $x = 0$ to find a, then substitute the other point and solve for b, keeping the positive root. To solve an equation like $5^{2x - 1} = 125$, rewrite both sides with the same base and use the **one-to-one property**: if $b^m = b^n$, then $m = n$.

The parent graph $y = b^x$ passes through (0, 1) and (1, b), has domain $(-\infty, \infty)$, range $(0, \infty)$ and the **horizontal asymptote** $y = 0$. It has no x-intercept. $(1/2)^x = 2^{-x}$, so decay is growth reflected across the y-axis.

In $g(x) = a \cdot b^{x - h} + k$, only k moves the asymptote, to $y = k$. The range becomes (k, ∞) if $a > 0$ and $(-\infty, k)$ if $a < 0$. Shifting right or left and stretching leave the asymptote alone. Draw the asymptote first, then plot three points. A vertical shift can create an x-intercept the parent did not have.

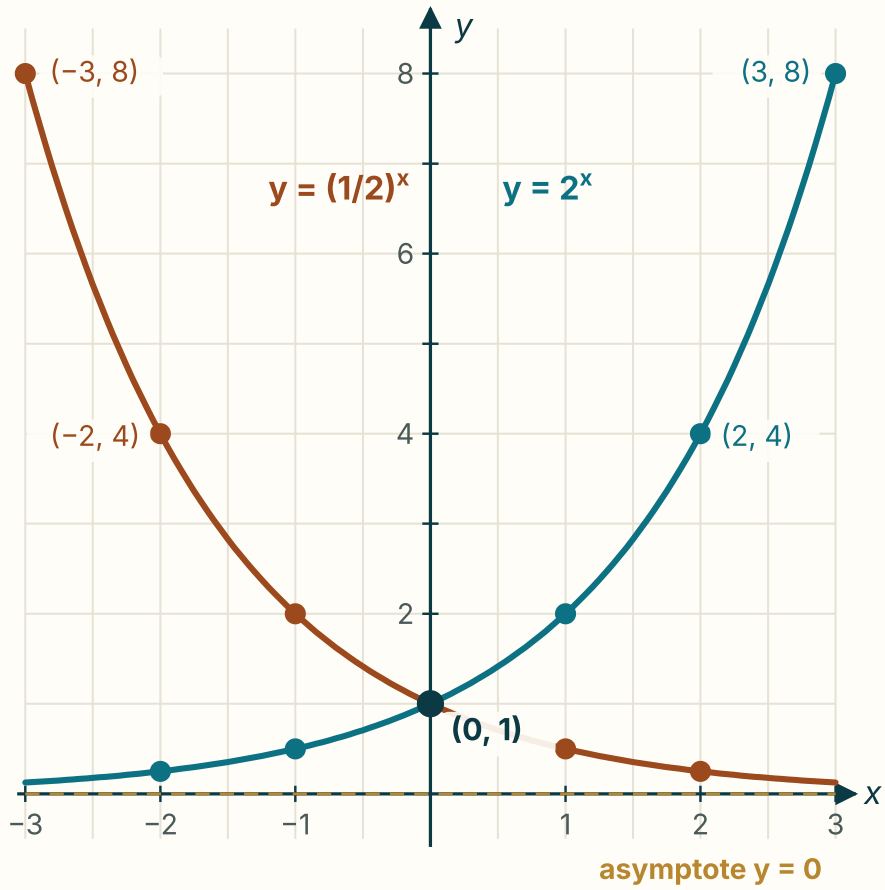
The number **e** ≈ 2.71828 is the value $(1 + 1/n)^n$ approaches as n grows: compounding more and more often earns more, but not without limit. $f(x) = e^x$ is the **natural exponential function**, an ordinary growth curve between 2^x and 3^x. Continuous change follows $A(t) = A_0e^{kt}$, with $k > 0$ for growth and $k < 0$ for decay. Simplify the exponent first and round only at the end.

Percent rates become bases: $A(t) = A_0(1 + r)^t$, so 3% growth has base 1.03 and 15% decay has base 0.85. A quantity that doubles every d periods follows $A_0 \cdot 2^{t/d}$, and one with **half-life** h follows $A_0 \cdot (1/2)^{t/h}$. The exponent t/h does not have to be a whole number. Percent change never adds across periods: losing 10% and then gaining 10% leaves you below where you started.

Compound interest is $A = P(1 + r/n)^{nt}$, with r as a decimal, n compoundings per year and t in years. Continuous compounding is $A = Pe^{rt}$, the ceiling the others approach. The **effective annual yield** is the one-year growth factor minus 1: 6% compounded monthly is really about 6.17%. Do not round r/n early.

LOCK THESE IN

- $f(x) = a \cdot b^x$, $a \neq 0$, $b > 0$, $b \neq 1$; $f(0) = a$.
- Each step right multiplies by b. $b > 1$ grows; $0 < b < 1$ decays.
- Parent: through (0, 1), range $(0, \infty)$, asymptote $y = 0$.
- In $a \cdot b^{x-h} + k$, only k moves the asymptote.
- $e \approx 2.71828$; $A = A_0e^{kt}$ for continuous change.
- Percent to base: $1 + r$. Doubling: $2^{t/d}$. Half-life: $(1/2)^{t/h}$.
- $A = P(1 + r/n)^{nt}$; continuous $A = Pe^{rt}$.
- Never multiply a percent by the number of years.



$f(x) = a \cdot b^x$, $b > 0$, $b \neq 1$

$b > 1$: growth

$0 < b < 1$: decay

each step right multiplies by b

both pass through (0, 1)

domain $(-\infty, \infty)$

range $(0, \infty)$

no x-intercept

$(1/2)^x = 2^{-x}$: a mirror image across the y-axis

The base decides growth or decay. Each step to the right multiplies the output by b. Both curves pass through (0, 1), stay above y = 0, and are mirror images of each other across the y-axis.

Exponential models and when to use each

Model	Formula	When to use it
General	$f(x) = a \cdot b^x$	Constant factor per step
Percent change	$A_0(1 + r)^t$	Given a percent per period; $r < 0$ for decay
Doubling time	$A_0 \cdot 2^{t/d}$	Given the time to double
Half-life	$A_0 \cdot (1/2)^{t/h}$	Given the time to halve
Continuous	A_0e^{kt}	Smooth, continuous change at rate k
Compound interest	$P(1 + r/n)^{nt}$	n compoundings per year
Continuous interest	Pe^{rt}	Compounded continuously
Effective yield	$(1 + r/n)^n - 1$ or $e^r - 1$	Comparing accounts fairly
Transformed graph	$a \cdot b^{x - h} + k$	Asymptote $y = k$
One-to-one property	$b^m = b^n$ gives $m = n$	Bases can be matched

How compounding frequency changes $5,000 at 6% for 10 years

Compounding	n	Balance
Annually	1	$8,954.24
Quarterly	4	$9,070.09
Monthly	12	$9,096.98
Daily	365	$9,110.14
Continuously	none	$9,110.59

Worked examples

WORKED EXAMPLE 1 · EVALUATING AN EXPONENTIAL

Let $f(x) = 2 \cdot 3^x$. Find $f(2)$, $f(0)$ and $f(-2)$.

STEP 1 · EXPONENT FIRST

$f(2) = 2 \cdot 9 =$ **18**. It is not $(2 \cdot 3)^2 = 36$.

STEP 2 · ZERO EXPONENT

$f(0) = 2 \cdot 1 =$ **2**, the initial value.

STEP 3 · NEGATIVE EXPONENT

$3^{-2} = 1/9$, so $f(-2) =$ **2/9** ≈ 0.222. Small, but still positive.

WORKED EXAMPLE 2 · BUILDING A MODEL FROM TWO POINTS

An exponential passes through (0, 4) and (2, 100). Find $f(x) = a \cdot b^x$.

STEP 1 · USE X = 0

$4 = a \cdot b^0 = a$, so $a = 4$.

STEP 2 · USE THE SECOND POINT

$100 = 4b^2$, so $b^2 = 25$.

STEP 3 · KEEP THE POSITIVE ROOT

$b = 5$, since the base must be positive. **$f(x) = 4 \cdot 5^x$**.

CHECK

$4 \cdot 5^2 = 4 \cdot 25 = 100$.

WORKED EXAMPLE 3 · A TRANSFORMED EXPONENTIAL

Describe $g(x) = 2 \cdot 3^{x-1} + 4$.

STEP 1 · READ THE PIECES

Right 1, stretch by 2, up 4.

STEP 2 · ASYMPTOTE AND RANGE

Only k moves the asymptote: **$y = 4$**. $a = 2 > 0$, so the range is **$(4, \infty)$**.

STEP 3 · THREE POINTS

$g(1) = 2 \cdot 1 + 4 = 6$; $g(2) = 2 \cdot 3 + 4 = 10$; $g(0) = 2/3 + 4 = 14/3$, the y-intercept.

NO X-INTERCEPT

The curve stays above y = 4, so it never reaches the x-axis.

WORKED EXAMPLE 4 · HALF-LIFE WITH A FRACTION OF A HALF-LIFE

A drug has a half-life of 5 hours. A 120 mg dose is given. How much remains after 15 hours? After 12 hours?

STEP 1 · MODEL

$A(t) = 120 \cdot (1/2)^{t/5}$.

STEP 2 · 15 HOURS

$t/h = 3$, so $120 \cdot 1/8 =$ **15 mg**. Halving: 120, 60, 30, 15.

STEP 3 · 12 HOURS

$t/h = 2.4$, so $120/2^{2.4} = 120/5.278 \approx$ **22.74 mg**.

IS IT BELIEVABLE?

At 10 hours there would be 30 mg and at 15 hours 15 mg; 22.74 mg sits between them.

WORKED EXAMPLE 5 · QUARTERLY VERSUS CONTINUOUS COMPOUNDING

Deposit $4,000 at 4.5% for 8 years. Compare quarterly and continuous compounding.

STEP 1 · QUARTERLY PIECES

$r/n = 0.045/4 = 0.01125$ and $nt = 32$.

STEP 2 · QUARTERLY BALANCE

$4000(1.01125)^{32} \approx 4000 \times 1.43045 =$ **$5,721.81**.

STEP 3 · CONTINUOUS

$rt = 0.36$, so $4000e^{0.36} \approx 4000 \times 1.433329 =$ **$5,733.32**.

STEP 4 · COMPARE

Continuous earns $11.51 more over 8 years: real, but small.

TRAPS PEOPLE FALL INTO

- **Confusing x^2 with 2^x.** Check whether the variable is the base or the exponent.
- **Multiplying before the exponent.** $3 \cdot 2^3 = 24$, not $6^3 = 216$.
- **Expecting a negative output from a negative exponent.** $2^{-2} = 1/4$.
- **Moving the asymptote with a horizontal shift.** Only the added k moves it.
- **Using the rate as the base.** 3% growth has base 1.03, not 0.03.

- **Adding percents over time.** 15% a year for 5 years leaves about 44%, not 25%.
- **Rounding r/n or e early.** Keep full precision until the final answer.
- **Forgetting that a half-life count can be fractional.** t/h = 2.4 is allowed.

Chapter 11 self-test · ten items

1. Only one of these meets every rule for $f(x) = a \cdot b^x$. Which?

A. $f(x) = 4x^{0.7}$

B. $f(x) = (-2)^x$

C. $f(x) = 4(1)^x$

D. $f(x) = 4(0.7)^x$

D. $f(x) = 4(0.7)^x$ The base 0.7 is positive and not 1, and x is in the exponent. A negative base or a base of 1 is not allowed.

2. Let $f(x) = 5 \cdot 2^x$. What is $f(-3)$?

A. −40

B. 1/40

C. 5/8

D. −5/8

C. 5/8 $2^{-3} = 1/8$, so $5 \cdot 1/8 = 5/8$. A negative exponent means reciprocal.

3. Find $a \cdot b^x$ through the points (0, 3) and (2, 48).

A. $f(x) = 3 \cdot 8^x$

B. $f(x) = 3 \cdot 4^x$

C. $f(x) = 4 \cdot 3^x$

D. $f(x) = 3 \cdot 16^x$

B. $f(x) = 3 \cdot 4^x$ $a = 3$. Then $48 = 3b^2$, so $b^2 = 16$ and $b = 4$.

4. What does $y = 9(1.06)^x$ describe?

A. Growth of 6% per step

B. Growth of 9% per step

C. Decay of 6% per step

D. Growth of 106% per step

A. Growth of 6% per step The base is 1 + r = 1.06, so r = 0.06. The 9 is the starting value.

5. Which line does $y = 4^{x+3} - 2$ flatten toward on the left?

A. y = 3

B. y = −2

C. y = −3

D. y = 0

B. y = −2 Only the added constant moves the asymptote. The +3 inside shifts the graph left.

6. Which outputs can $h(x) = -2 \cdot 3^x + 5$ produce?

A. (−2, ∞)

B. (−∞, −2)

C. (−∞, 5)

D. (5, ∞)

C. (−∞, 5) The asymptote is y = 5, and the negative coefficient puts the curve below it.

7. Solve $2^{x-1} = 64$.

A. x = 7

B. x = 6

C. x = 32

D. x = 5

A. x = 7 $64 = 2^6$, so $x - 1 = 6$ and $x = 7$.

8. $1,500 grows continuously at 4% a year for 10 years. About how much is there?

A. $2,100.00

B. $2,220.37

C. $1,560.00

D. $2,237.74

D. $2,237.74 $A = 1500e^{0.4} \approx 1500 \times 1.4918 = 2,237.74$. Simple interest would give $2,100.

9. An isotope halves every 3 days. Start with 64 g; after 12 days what is left?

A. 2 g

B. 4 g

C. 16 g

D. 8 g

B. 4 g 12/3 = 4 half-lives: 64, 32, 16, 8, 4.

10. $1,000 earns 6% compounded monthly for 2 years. What is the balance?

A. $1,123.60
B. $1,120.00
C. $1,012.00
D. $1,127.16

D. $1,127.16 $1000(1.005)^{24} \approx 1{,}127.16$. Annual compounding would give $1,123.60.

CHAPTER 12

Logarithmic Functions

Read it in full: **Chapter 12 in the textbook**. Practice it in the lab: **A Log Is an Exponent** and **Expand and Condense**.

What to know

A **logarithm** is an exponent. $\log_b(x) = y$ means exactly $b^y = x$: y is the power you put on b to get x. Because exponential functions are one-to-one, they have inverses, and the logarithm is that inverse: $\log_b(b^x) = x$ and $b^{\log_b(x)} = x$.

To convert, the base stays the base and the log's answer becomes the exponent: $2^5 = 32$ is $\log_2(32) = 5$. The base must be positive and not 1. The **argument** must be positive, because a positive base raised to any power is positive, so $\log_2(-8)$ is undefined. To solve a simple log equation, convert to exponential form, solve, and check that the argument is positive.

log(x) with no base is the **common logarithm**, base 10. ln(x) is the **natural logarithm**, base e. On a calculator they are different buttons. Four facts come straight from the definition: $\log_b(1) = 0$, $\log_b(b) = 1$, $\log_b(b^k) = k$ and $b^{\log_b(x)} = x$. Reciprocal arguments give negative answers and roots give fractional ones.

To evaluate by inspection, set the log equal to y, convert, rewrite both sides with a common base and match exponents: $\log_8(4) = 2/3$ because $8^{2/3} = 4$. Estimate first: $\log_7(200)$ is between 2 and 3

because $49 < 200 < 343$.

The parent graph $y = \log_b(x)$ has domain $(0, \infty)$, range all reals, the **vertical asymptote** $x = 0$, the x-intercept $(1, 0)$ and the point $(b, 1)$. It rises ever more slowly: to go up one unit you must multiply x by b. Every feature of $y = b^x$ is swapped, because the two graphs are mirror images across $y = x$.

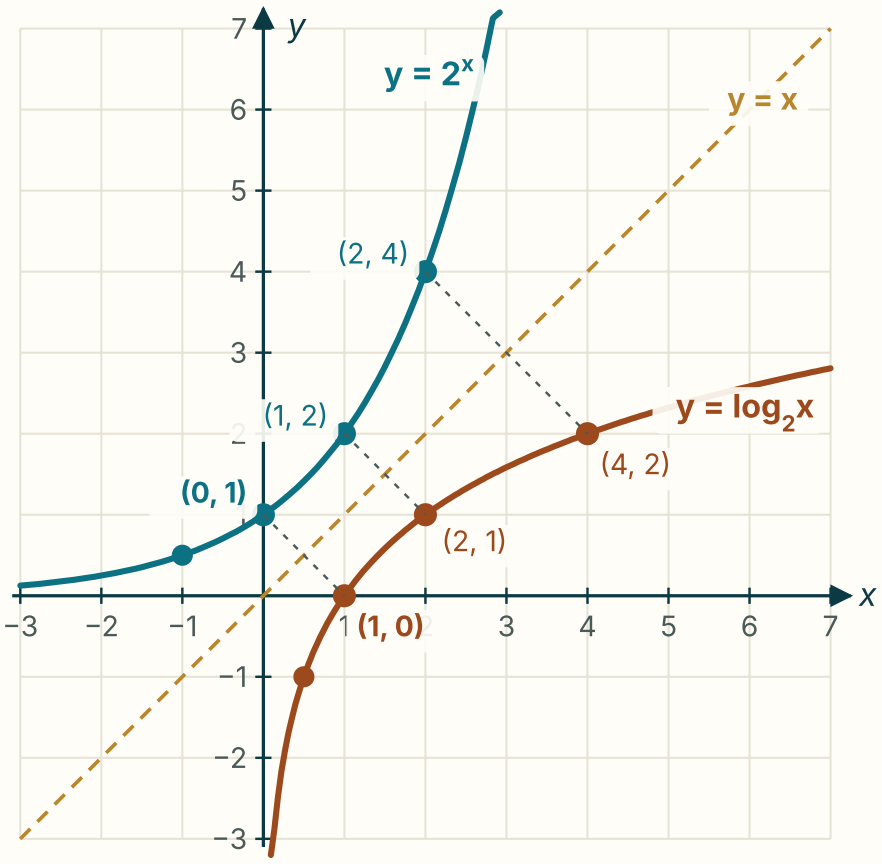
In $y = a \cdot \log_b(x - h) + k$, only h moves the asymptote, to $x = h$. Find the domain by setting the argument > 0, and the asymptote by setting it $= 0$. Dividing that inequality by a negative flips it: $\log(8 - 2x)$ needs $x < 4$. A horizontal shift can give the graph a y-intercept the parent did not have.

Three properties, each the twin of an exponent rule: **product** $\log_b(MN) = \log_b M + \log_b N$, **quotient** $\log_b(M/N) = \log_b M - \log_b N$, and **power** $\log_b(M^p) = p \cdot \log_b M$. Three misuses lose the most points: a sum inside a log cannot be split, a ratio of two logs is not the log of a quotient, and $(\log M)^2$ is not $2 \log M$.

To expand, apply quotient, then product, then power, and evaluate any plain numbers. Rewrite radicals as fractional exponents first. To condense, reverse the order: power, then product, then quotient, ending with one log of coefficient 1. **Change of base**: $\log_b M = \ln M / \ln b = \log M / \log b$, with the old base on the bottom. When an equation has two logs, condense, convert, solve, and discard any candidate that makes an original argument zero or negative.

LOCK THESE IN

- $\log_b(x) = y$ means $b^y = x$. A log is an exponent.
- Base: positive, not 1. Argument: positive.
- log is base 10; ln is base e.
- $\log_b 1 = 0$, $\log_b b = 1$, $\log_b b^k = k$.
- Parent: domain $(0, \infty)$, VA $x = 0$, through $(1, 0)$ and $(b, 1)$.
- Domain: argument > 0. Asymptote: argument $= 0$.
- Product to sum, quotient to difference, power to coefficient.
- Change of base puts the old base on the bottom; check for extraneous answers.



$\log_b x = y$ means $b^y = x$

a log is an exponent

swap every (x, y):

(0, 1) becomes (1, 0)

(2, 4) becomes (4, 2)

exponential:

domain all reals

range (0, ∞), HA y = 0

logarithm:

domain (0, ∞)

range all reals, VA x = 0

Inverse functions mirror across y = x. Swapping each point of $y = 2^x$ gives a point of $y = \log_2 x$. Domain and range trade places, and the horizontal asymptote y = 0 becomes the vertical asymptote x = 0.

Logarithm rules and when to use them

Rule	Formula	When to use it
Definition	$\log_b x = y \Leftrightarrow b^y = x$	Converting forms; simple equations
Inverse pair	$\log_b(b^x) = x$; $b^{\log_b x} = x$	Canceling a log against its base
Special values	$\log_b 1 = 0$; $\log_b b = 1$	Evaluating by sight
Product	$\log_b(MN) = \log_b M + \log_b N$	Expanding or condensing
Quotient	$\log_b(M/N) = \log_b M - \log_b N$	Expanding or condensing
Power	$\log_b(M^p) = p \log_b M$	Bringing exponents out front
Change of base	$\log_b M = \ln M / \ln b$	Any base on a calculator
Domain	Argument > 0	Every log function and equation
Transformed graph	$a \log_b(x - h) + k$	Asymptote $x = h$; domain $x > h$

Exponential versus logarithmic graphs

Feature	$y = b^x$	$y = \log_b x$
Domain	$(-\infty, \infty)$	$(0, \infty)$
Range	$(0, \infty)$	$(-\infty, \infty)$
Asymptote	$y = 0$	$x = 0$
Key points	(0, 1), (1, b)	(1, 0), (b, 1)

Worked examples

WORKED EXAMPLE 1 · A SIMPLE LOG EQUATION

Solve $\log_3(4x + 1) = 2$.

STEP 1 · CONVERT

$3^2 = 4x + 1$, so $9 = 4x + 1$.

STEP 2 · SOLVE

$4x = 8$, so **$x = 2$**.

STEP 3 · CHECK THE ARGUMENT

$4(2) + 1 = 9 > 0$, and $\log_3 9 = 2$ because $3^2 = 9$.

WORKED EXAMPLE 2 · EVALUATING BY INSPECTION

Evaluate $\log_4(8)$, $\log_2(1/16)$ and $\log(0.001)$.

STEP 1 · COMMON BASE

$4^y = 8$ becomes $2^{2y} = 2^3$, so $y =$ **$3/2$**.

STEP 2 · RECIPROCAL

$2^{-4} = 1/16$, so $\log_2(1/16) =$ **-4**.

STEP 3 · BASE 10

$0.001 = 10^{-3}$, so $\log(0.001) =$ **-3**.

CHECK THE FIRST

$4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$.

WORKED EXAMPLE 3 · A SHIFTED LOG GRAPH

For $f(x) = \log_3(x - 2) + 1$, find the domain, asymptote, x-intercept and two more points.

STEP 1 · DOMAIN AND ASYMPTOTE

$x - 2 > 0$, so the domain is **$(2, \infty)$** and the asymptote is **$x = 2$**.

STEP 2 · X-INTERCEPT

$\log_3(x - 2) = -1$, so $x - 2 = 1/3$ and $x =$ **$7/3$**.

STEP 3 · MORE POINTS

$f(3) = 0 + 1 = 1$, so $(3, 1)$. $f(5) = 1 + 1 = 2$, so $(5, 2)$.

RANGE

Still all real numbers; a vertical shift never limits it.

WORKED EXAMPLE 4 · USING THE PROPERTIES WITH GIVEN VALUES

Given $\log 2 \approx 0.3010$ and $\log 3 \approx 0.4771$, find $\log 18$.

STEP 1 · FACTOR

$18 = 2 \cdot 3^2$.

STEP 2 · PRODUCT AND POWER RULES

$\log 18 = \log 2 + 2 \log 3$.

STEP 3 · SUBSTITUTE

$0.3010 + 2(0.4771) = 0.3010 + 0.9542 =$ **1.2552**.

REASONABLE?

18 is between 10 and 100, so the log is between 1 and 2.

WORKED EXAMPLE 5 · EXPAND, THEN CONDENSE

Expand $\log_2(16\sqrt{x}/y^3)$. Then condense $\ln x - 2 \ln(x + 1) + 3 \ln y$.

STEP 1 · QUOTIENT, PRODUCT, POWER

$\log_2 16 + \log_2 x^{1/2} - \log_2 y^3 =$ **$4 + \frac{1}{2} \log_2 x - 3 \log_2 y$**.

STEP 2 · CONDENSE: POWER FIRST

$\ln x - \ln(x + 1)^2 + \ln y^3$.

STEP 3 · PRODUCT, THEN QUOTIENT

$\ln[xy^3/(x + 1)^2]$.

WORKED EXAMPLE 6 · AN EQUATION WITH AN EXTRANEOUS CANDIDATE

Solve $\log_3(x) + \log_3(x + 6) = 3$.

STEP 1 · DOMAIN

$x > 0$ and $x + 6 > 0$, so $x > 0$.

STEP 2 · CONDENSE AND CONVERT

$\log_3[x(x + 6)] = 3$, so $x^2 + 6x = 27$.

STEP 3 · SOLVE

$x^2 + 6x - 27 = 0 = (x + 9)(x - 3)$, so $x = -9$ or $x = 3$.

STEP 4 · SCREEN

$x = -9$ is extraneous. **$x = 3$**: $\log_3 3 + \log_3 9 = 1 + 2 = 3$.

TRAPS PEOPLE FALL INTO

- **Treating log as a mystery operation.** Ask "b to what power gives x?"

- **Taking the log of zero or a negative.** The argument must be positive.
- **Pressing LN for log.** LOG is base 10; LN is base e.
- **Saying the asymptote is always x = 0.** Set the argument equal to zero; a horizontal shift moves it.
- **Splitting a sum inside a log.** $\log(x + 5) \neq \log x + \log 5$.
- **Treating a ratio of logs as a log of a ratio.** $\log_2 8/\log_2 2 = 3$, but $\log_2 4 = 2$.
- **Putting the old base on top in change of base.** $\log_5 89 = \ln 89/\ln 5$.
- **Keeping every candidate.** Discard any value that makes an original argument zero or negative.

Chapter 12 self-test · ten items

1. Which is $5^3 = 125$ in logarithmic form?

A. $\log_5(3) = 125$

B. $\log_{125}(5) = 3$

C. $\log_3(125) = 5$

D. $\log_5(125) = 3$

D. $\log_5(125) = 3$ The base stays the base and the exponent becomes the log's answer.

2. Evaluate $\log_6(1/36)$.

A. 1/2

B. −2

C. −1/2

D. 2

B. −2 $6^{-2} = 1/36$. A reciprocal argument gives a negative answer.

3. 27 raised to what power gives 3? That is, find $\log_{27}(3)$.

A. 9

B. −3

C. 1/3

D. 3

C. 1/3 $27^{1/3} = \sqrt[3]{27} = 3$. A root gives a fractional answer.

4. Solve $\log_2(x) + \log_2(x + 2) = 3$.

A. x = 2 only

B. x = −4 only

C. x = 2 and x = −4

D. No real value

A. x = 2 only $x(x + 2) = 8$ gives $(x + 4)(x − 2) = 0$. x = −4 makes the arguments negative, so it is extraneous.

5. Which inputs keep the argument of log(5 − x) positive?

A. (−5, ∞)

B. (−∞, 5]

C. (−∞, 5)

D. (5, ∞)

C. (−∞, 5) $5 − x > 0$ gives $−x > −5$; dividing by −1 flips it to $x < 5$.

6. For $y = \log_2(x + 4)$, what are the asymptote and x-intercept?

A. $x = 4$ and $(5, 0)$

B. $x = -4$ and $(-3, 0)$

C. $x = 0$ and $(-3, 0)$

D. $x = -4$ and $(1, 0)$

B. $x = -4$ and $(-3, 0)$ The argument is zero at $x = -4$. For the intercept, $x + 4 = 2^0 = 1$, so $x = -3$.

7. By the product rule, $\log_b(MN)$ can be rewritten as which?

A. $\log_b(M + N)$

B. $N \cdot \log_b M$

C. $\log_b M \cdot \log_b N$

D. $\log_b M + \log_b N$

D. $\log_b M + \log_b N$ Multiplication inside becomes addition outside: the product rule.

8. Expand $\log(x^2y/10)$.

A. $2 \log x + \log y - 1$

B. $2 \log x + \log y + 1$

C. $2 \log x - \log y - 1$

D. $\log 2x + \log y - 1$

A. $2 \log x + \log y - 1$ Quotient, product, power: $\log x^2 + \log y - \log 10$, and $\log 10 = 1$.

9. Condense $3 \ln x - \ln y$ into a single logarithm.

A. $\ln(3x/y)$

B. $\ln(x^3 - y)$

C. $\ln(x^3/y)$

D. $\ln(x^3 y)$

C. $\ln(x^3/y)$ Power first: $\ln x^3 - \ln y$. Then the quotient rule gives $\ln(x^3/y)$.

10. Use change of base to evaluate $\log_4(30)$.

A. About 2.4534

B. About 1.4771

C. About 7.5000

D. About 0.4076

A. About 2.4534 $\ln 30/\ln 4 \approx 3.4012/1.3863 \approx 2.4534$. It lies between 2 and 3, since $16 < 30 < 64$.

ONE PAGE

Cram sheet

Everything from chapters 11 and 12 in one place. Cover the right column and see if you can rebuild each row.

Exponential formulas

Name	Formula	When to use it
Exponential function	$a \cdot b^x$, $b > 0$, $b \neq 1$	Constant factor per step
Transformed	$a \cdot b^{x-h} + k$	Asymptote $y = k$
Percent	$A_0(1 + r)^t$	Rate per period
Doubling and half-life	$A_0 2^{t/d}$; $A_0(1/2)^{t/h}$	Given d or h
Continuous	$A_0 e^{kt}$; Pe^{rt}	Smooth change; continuous interest
Compound interest	$P(1 + r/n)^{nt}$	n times a year
Effective yield	$(1 + r/n)^n - 1$	Comparing accounts
e	$\lim (1 + 1/n)^n \approx 2.71828$	Natural base

Logarithm rules

Name	Rule	Note
Definition	$\log_b x = y \Leftrightarrow b^y = x$	A log is an exponent
Special values	$\log_b 1 = 0$; $\log_b b = 1$; $\log_b b^k = k$	Read by sight
Product	$\log(MN) = \log M + \log N$	Not $\log(M + N)$
Quotient	$\log(M/N) = \log M - \log N$	Not $\log M / \log N$
Power	$\log(M^p) = p \log M$	Not $(\log M)^p$
Change of base	$\log_b M = \ln M / \ln b$	Old base on the bottom
Domain	Argument > 0	Asymptote where argument = 0

Graph facts

Graph	Domain	Range	Asymptote	Key points
$y = b^x$	$(-\infty, \infty)$	$(0, \infty)$	$y = 0$	(0, 1), (1, b)
$y = \log_b x$	$(0, \infty)$	$(-\infty, \infty)$	$x = 0$	(1, 0), (b, 1)

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