COLLEGE ALGEBRA · STUDY GUIDE

Exam 5 Study Guide

This guide covers chapters 9 and 10. You will solve quadratics four ways, read the discriminant, graph parabolas from the vertex out and set up projectile and area problems. Then you will read a polynomial's ends and zeros, divide with synthetic division and build rational graphs from their holes and asymptotes. Work one chapter at a time. Read the "what to know" paragraphs, copy the tables out by hand, then try each worked example with the steps covered. Finish each chapter with its ten self-test items. The cram sheet at the end puts every formula in one place.

WHAT THIS GUIDE COVERS

- Chapter 9 · the zero product property, the square root property, completing the square, the quadratic formula and discriminant, graphing parabolas and vertex form, and applications.
- Chapter 10 · degree and end behavior, zeros and multiplicity, long and synthetic division, the Remainder and Factor Theorems, rational function domains, holes and asymptotes, and full rational graphs.

CHAPTER 9

Quadratic Equations and Functions

Read it in full: **Chapter 9 in the textbook**. Practice it in the lab: **The Missing Corner** and **The Number Under the Radical**.

What to know

A **quadratic equation** can be written $ax^2 + bx + c = 0$ with $a \neq 0$. That arrangement, with 0 on one side, is **standard form**, and almost every method starts there. The **zero product property** says that if $A \cdot B = 0$, then $A = 0$ or $B = 0$. It works only against a zero: $(x + 2)(x - 5) = 18$ must be multiplied out and rearranged first.

Solving by factoring: standard form, factor completely, set each factor to zero, solve, check. Pull out a common factor first, and never divide both sides by x, because that throws away $x = 0$. A

repeated factor such as $(x - 3)^2$ gives a **double root**: one solution.

The **square root property**: if $x^2 = k$, then $x = \pm\sqrt{k}$. The ± is mandatory, and $k < 0$ means no real solution. It works on any squared expression: $(x - 3)^2 = 49$ gives $x - 3 = \pm 7$. Simplify the radical: $x^2 = 12$ gives $x = \pm 2\sqrt{3}$.

Completing the square manufactures a perfect square. Make the leading coefficient 1, move the constant, add $(b/2)^2$ to both sides, factor the left side as $(x + b/2)^2$, then use the square root property. It works on every quadratic, including ones that do not factor, and it is where the quadratic formula comes from.

The **quadratic formula** $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$ solves every quadratic. Put the equation in standard form, read a, b and c with their signs, and divide the whole numerator by 2a. −b means the opposite of b, so $b = -5$ gives +5. The **discriminant** $b^2 - 4ac$ predicts the outcome: positive gives two real solutions (rational if it is a perfect square), zero gives one double root, and negative gives none in the real numbers.

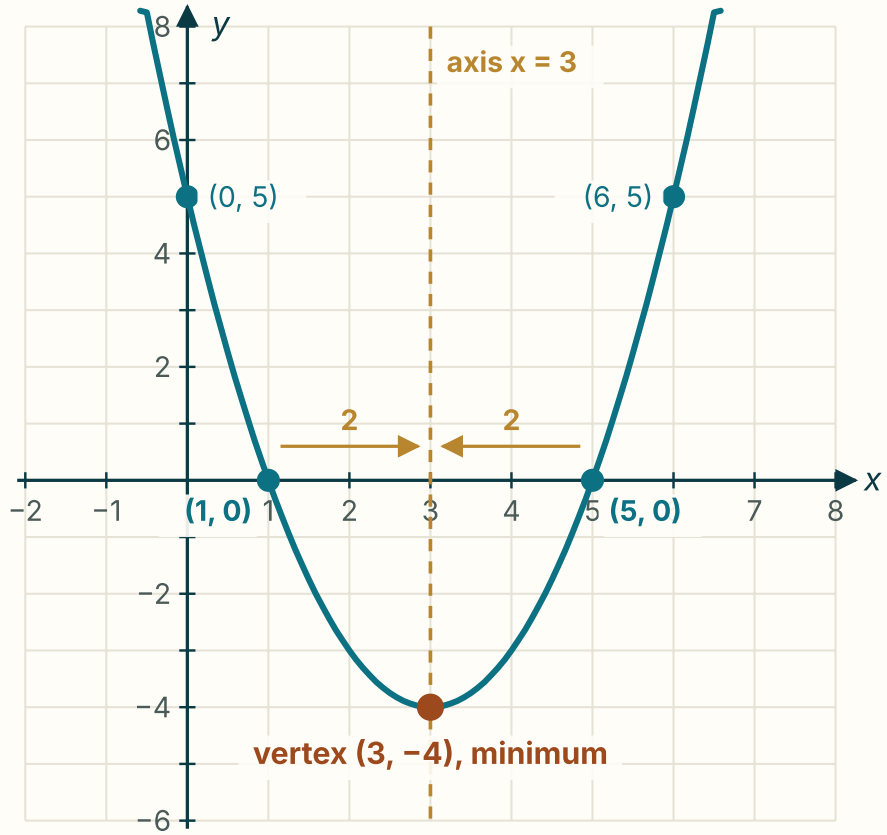
The graph of $f(x) = ax^2 + bx + c$ is a **parabola**. If $a > 0$ it opens up and the vertex is a minimum; if $a < 0$ it opens down and the vertex is a maximum. Larger $|a|$ makes it narrower. The **axis of symmetry** is $x = -b/(2a)$, and the vertex is that x with f of that x. The y-intercept is (0, c), and the x-intercepts are the real solutions of $f(x) = 0$. The axis always sits halfway between the roots.

Vertex form $f(x) = a(x - h)^2 + k$ shows the vertex (h, k) at once. Watch the sign: $2(x + 3)^2 - 7$ has $h = -3$. Convert by completing the square. The domain of every quadratic is all real numbers; the range is $y \geq k$ if it opens up and $y \leq k$ if it opens down.

Applications ask one of two questions. A vertex question asks how high, how large or how much: use $x = -b/(2a)$, then substitute. An equation question asks when or what size: set the function equal to the value and solve, often getting two answers. Projectile height in feet and seconds is $h(t) = -16t^2 + v_0t + h_0$. Revenue is price times quantity. Reject answers the story forbids, such as negative time or length, and answer in the units asked.

LOCK THESE IN

- Standard form first: $ax^2 + bx + c = 0$.
- Zero product property works only against 0.
- Never divide by x; factor it out instead.
- $x^2 = k$ gives $x = \pm\sqrt{k}$.
- Complete the square: add $(b/2)^2$ to both sides.
- $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$; the discriminant counts real solutions.
- Axis $x = -b/(2a)$; vertex form $a(x - h)^2 + k$.
- In word problems, keep only answers the situation allows.



f(x) = x² − 6x + 5

a = 1 > 0: opens up

x = −b/(2a) = 6/2 = 3

f(3) = 9 − 18 + 5 = −4

roots: (x − 1)(x − 5)

average of roots = 3

vertex form:

(x − 3)² − 4

discriminant:

36 − 20 = 16 > 0

two real roots

Vertex, axis, intercepts. The axis $x = -b/(2a) = 3$ sits halfway between the roots 1 and 5, a free check. Every point has a mirror image across it, such as (0, 5) and (6, 5).

Quadratic formulas and when to use them

Tool	Formula	When to use it
Zero product property	$AB = 0$ gives $A = 0$ or $B = 0$	The quadratic factors easily
Square root property	$x^2 = k$ gives $x = \pm\sqrt{k}$	Form $(x - h)^2 = k$, or no x term
Completing the square	$x^2 + bx + (b/2)^2 = (x + b/2)^2$	Vertex form; small even b
Quadratic formula	$x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$	Any quadratic, especially ones that do not factor
Discriminant	$D = b^2 - 4ac$	$D > 0$: two; $D = 0$: one; $D < 0$: none real
Axis and vertex	$x = -b/(2a)$; vertex $(-b/(2a), f(-b/(2a)))$	Graphing; max and min problems
Vertex form	$f(x) = a(x - h)^2 + k$	Reading the vertex (h, k)
Projectile	$h(t) = -16t^2 + v_0t + h_0$	Feet and seconds under gravity
Revenue	$R = (\text{price})(\text{quantity})$	Best price for maximum revenue

What the discriminant tells you

$b^2 - 4ac$	Real solutions	Graph	Example
Positive square	Two, rational	Crosses twice	$2x^2 + 5x - 3$ (49)
Positive, not a square	Two, irrational	Crosses twice	$3x^2 - 6x + 2$ (12)
Zero	One double root	Touches at the vertex	$4x^2 - 12x + 9$ (0)
Negative	None	Never meets the axis	$x^2 + x + 3$ (−11)

Worked examples

WORKED EXAMPLE 1 · SOLVING BY FACTORING

Solve $x^2 - 2x = 15$, then $4x^2 = 12x$.

STEP 1 · STANDARD FORM

$x^2 - 2x - 15 = 0$.

STEP 2 · FACTOR

Product −15, sum −2: −5 and 3. $(x - 5)(x + 3) = 0$, so **$x = 5$ or $x = -3$**.

STEP 3 · SECOND EQUATION: MOVE EVERYTHING LEFT

$4x^2 - 12x = 0$, so $4x(x - 3) = 0$.

STEP 4 · ZERO PRODUCT PROPERTY

$x = 0$ or $x = 3$. Dividing by x at the start would have lost $x = 0$.

CHECK

$25 - 10 = 15$ and $9 + 6 = 15$. For the second: $0 = 0$ and $36 = 36$.

WORKED EXAMPLE 2 · COMPLETING THE SQUARE

Solve $x^2 - 8x + 3 = 0$ by completing the square.

STEP 1 · MOVE THE CONSTANT

$x^2 - 8x = -3$.

STEP 2 · HALVE AND SQUARE

Half of −8 is −4, and $(-4)^2 = 16$. Add 16 to both sides: $x^2 - 8x + 16 = 13$.

STEP 3 · FACTOR

$(x - 4)^2 = 13$.

STEP 4 · SQUARE ROOT PROPERTY

$x - 4 = \pm\sqrt{13}$, so **$x = 4 \pm \sqrt{13}$**, about 7.606 and 0.394.

CHECK ONE

$7.606^2 - 8(7.606) + 3 = 57.85 - 60.85 + 3 = 0$, to rounding.

WORKED EXAMPLE 3 · THE QUADRATIC FORMULA

Solve $2x^2 - 3x - 4 = 0$.

STEP 1 · READ THE COEFFICIENTS

$a = 2$, $b = -3$, $c = -4$.

STEP 2 · DISCRIMINANT

$(-3)^2 - 4(2)(-4) = 9 + 32 = 41$. Positive and not a square: two irrational roots.

STEP 3 · SUBSTITUTE

$x = (3 \pm \sqrt{41})/4$.

STEP 4 · DECIMALS

$\sqrt{41} \approx 6.403$, so **$x \approx 2.351$ or $x \approx -0.851$**.

WORKED EXAMPLE 4 · GRAPHING A DOWNWARD PARABOLA

Graph $f(x) = -x^2 + 4x + 5$.

STEP 1 · DIRECTION

$a = -1 < 0$, so it opens down and has a maximum.

STEP 2 · AXIS AND VERTEX

$x = -4/(2(-1)) = 2$. $f(2) = -4 + 8 + 5 = 9$. Vertex **(2, 9)**, maximum 9.

STEP 3 · INTERCEPTS

y-intercept (0, 5). Roots: $-(x^2 - 4x - 5) = -(x - 5)(x + 1) = 0$, so (5, 0) and (−1, 0).

STEP 4 · VERTEX FORM

$-(x - 2)^2 + 9$. Range $y \le 9$.

CHECK THE SYMMETRY

The roots −1 and 5 average to 2, the axis.

WORKED EXAMPLE 5 · A PROJECTILE

A ball is thrown up from a 96-foot roof: $h(t) = -16t^2 + 80t + 96$. Find the peak and the landing time.

STEP 1 · VERTEX TIME

$t = -80/(2(-16)) = 2.5$ seconds.

STEP 2 · PEAK HEIGHT

$h(2.5) = -16(6.25) + 200 + 96 = -100 + 296 =$ **196 feet**.

STEP 3 · SET HEIGHT TO 0

$-16t^2 + 80t + 96 = 0$. Divide by −16: $t^2 - 5t - 6 = 0$.

STEP 4 · FACTOR AND REJECT

$(t - 6)(t + 1) = 0$. Time cannot be negative, so the ball lands at **t = 6 seconds**.

TRAPS PEOPLE FALL INTO

- **Setting each factor equal to a nonzero number.** $(x + 2)(x - 5) = 18$ does not give $x + 2 = 18$.
- **Dividing both sides by x.** You lose the solution $x = 0$.

- **Forgetting the ±.** $x^2 = 49$ has two solutions, 7 and −7.
- **Adding $(b/2)^2$ to one side only.** Completing the square adds it to both sides.
- **Dividing only part of the numerator by 2a.** The whole $-b \pm \sqrt{D}$ sits over 2a.
- **Reading −b as negative.** If b = −5, then −b = +5.
- **Reading h with the wrong sign.** $(x + 3)^2$ means h = −3.
- **Reporting x when the question asks for a price.** Translate the answer back into the story.

Chapter 9 self-test · ten items

1. Factor to find both roots of $x^2 + 3x - 28 = 0$.

A. x = 7 or x = −4

B. x = 7 or x = 4

C. x = −7 or x = 4

D. x = −7 or x = −4

C. x = −7 or x = 4 Product −28 and sum 3: 7 and −4, so $(x + 7)(x - 4) = 0$.

2. Find every solution of $5x^2 = 20x$.

A. x = 4 only

B. x = 4 or x = 5

C. x = −4 or x = 0

D. x = 0 or x = 4

D. x = 0 or x = 4 $5x^2 - 20x = 0$ factors as $5x(x - 4) = 0$. Dividing by x would lose x = 0.

3. Use the square root property on $(x + 1)^2 = 36$.

A. $x = 5$ or $x = -7$

B. $x = -5$ or $x = 7$

C. $x = 5$ only

D. $x = 35$ or $x = -37$

A. $x = 5$ or $x = -7$ $x + 1 = \pm 6$, so $x = 5$ or $x = -7$.

4. To complete the square on $x^2 + 10x$, what number do you add?

A. 100

B. 25

C. 10

D. 5

B. 25 Half of 10 is 5, and $5^2 = 25$.

5. Use the discriminant: how many real roots does $3x^2 - 4x + 2 = 0$ have?

A. Two

B. One

C. Three

D. None

D. None The discriminant is $16 - 24 = -8$, which is negative.

6. Apply the quadratic formula to $x^2 + 2x - 4 = 0$.

A. $x = -1 \pm \sqrt{5}$

B. $x = -2 \pm \sqrt{5}$

C. $x = 1 \pm \sqrt{5}$

D. $x = -1 \pm \sqrt{20}$

A. $x = -1 \pm \sqrt{5}$ $x = (-2 \pm \sqrt{20})/2 = (-2 \pm 2\sqrt{5})/2 = -1 \pm \sqrt{5}$. Both terms of the numerator are divided by 2.

7. What is the vertex of $f(x) = 3x^2 - 12x + 7$?

A. (4, 7)

B. (−2, 43)

C. (2, −5)

D. (2, 7)

C. (2, −5) $x = 12/6 = 2$, and $f(2) = 12 - 24 + 7 = -5$.

8. Which is the vertex form of $x^2 - 4x + 1$?

A. $(x - 2)^2 + 1$

B. $(x - 2)^2 - 3$

C. $(x + 2)^2 - 3$

D. $(x - 4)^2 - 15$

B. $(x - 2)^2 - 3$ Add and subtract 4: $(x^2 - 4x + 4) - 4 + 1 = (x - 2)^2 - 3$.

9. A ball follows $h(t) = -16t^2 + 32t + 48$ feet. What is its maximum height?

A. 80 feet, at t = 2 s
B. 48 feet, at t = 0 s
C. 64 feet, at t = 1 s
D. 64 feet, at t = 2 s

C. 64 feet, at t = 1 s t = −32/(−32) = 1, and h(1) = −16 + 32 + 48 = 64.

10. A rectangle has perimeter 30 ft and area 56 ft^2. What are its sides?

A. 4 ft by 14 ft
B. 7 ft by 8 ft
C. 6 ft by 9 ft
D. 5 ft by 10 ft

B. 7 ft by 8 ft $x(15 - x) = 56$ gives $x^2 - 15x + 56 = 0 = (x - 7)(x - 8)$.

CHAPTER 10

Polynomial and Rational Functions

Read it in full: **Chapter 10 in the textbook**. Practice it in the lab: **Zeros and Multiplicity** and **Asymptotes and Holes**.

What to know

A **polynomial function** is a sum of terms $a \cdot x^n$ with whole-number exponents. 5/x and √x are not polynomial terms. Polynomial graphs are continuous (no gaps) and smooth (no corners). Write the terms from highest power down: the **degree** is the highest exponent and the **leading coefficient** is the number in front of it, sign included. Degree n allows at most n real zeros and at most n − 1 turning points.

End behavior depends only on the degree and the sign of the leading coefficient, because far from the origin the leading term dominates. Even degree: both ends agree, like x^2. Odd degree: the ends disagree, like x^3. A negative leading coefficient flips the picture. So $-2x^3 + \dots$ rises on the left and falls on the right.

A **zero** c makes P(c) = 0. Root, solution and x-intercept are other names for it. Find zeros by factoring completely, pulling out the GCF first, then using the zero product property, and check each one. The exponent on a repeated factor is its **multiplicity**. Odd multiplicity: the graph crosses. Even multiplicity: it touches and turns back. The multiplicities add up to the degree, and the leading term can be read from the factors without expanding.

Dividing polynomials follows dividend = divisor · quotient + remainder, with the remainder of lower degree than the divisor. Long division (divide, multiply, subtract, bring down) works for any divisor. **Synthetic division** works only for x − c: bring down, multiply by c, add. Use c = −5 for x + 5, and write a 0 for every missing power.

The **Remainder Theorem**: dividing P(x) by x − c leaves remainder P(c). The **Factor Theorem**: remainder 0 means (x − c) is a factor and c is a zero. The **Rational Zero Theorem** lists the candidates: ± factors of the constant over factors of the leading coefficient. Find one zero, divide it out, and factor the smaller quotient.

A **rational function** is P(x)/Q(x). Its domain excludes every zero of the original denominator, so state the domain before canceling. A factor that cancels leaves a **hole**; get its height from the simplified form. A factor that stays in the denominator gives a **vertical asymptote**, which the graph never crosses.

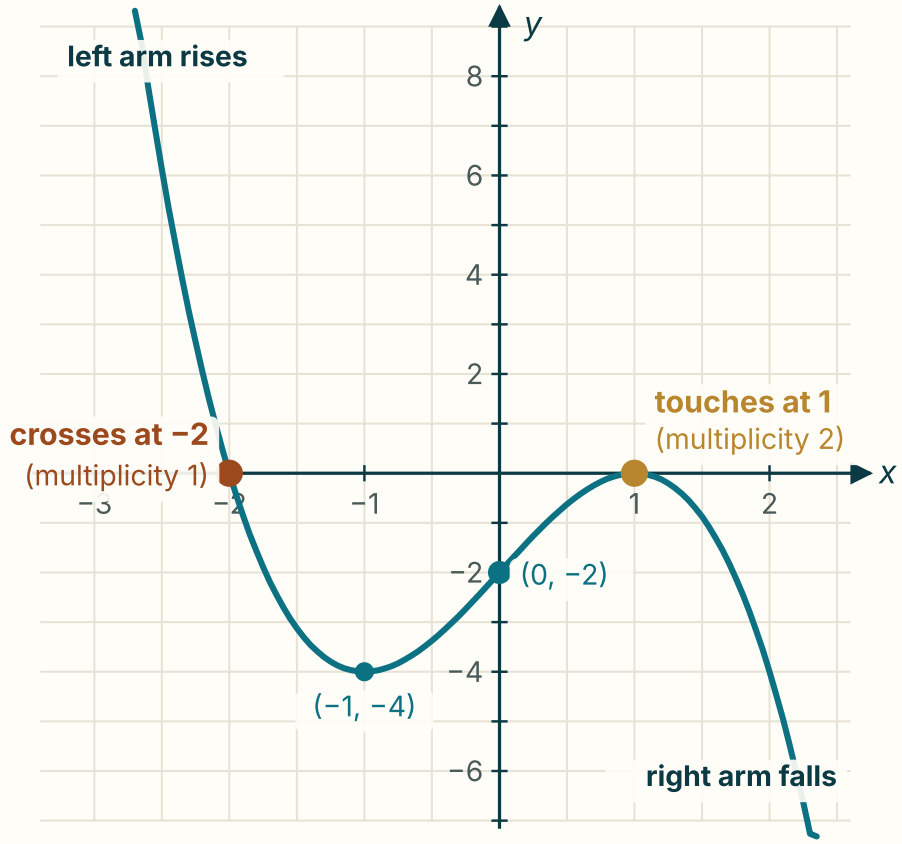
The **horizontal asymptote** compares degrees, top n and bottom m. If n < m, it is y = 0. If n = m, it is y = (ratio of leading coefficients). If n = m + 1, there is a **slant asymptote** found by division. If n > m + 1, there is neither. A graph may cross its horizontal asymptote in the middle, but never a vertical one.

To graph a rational function: factor, find the domain, cancel (holes), find vertical asymptotes, x-intercepts from the remaining numerator, the y-intercept f(0), the horizontal or slant asymptote, then a sign chart with one test value in each strip. Check the ends with x = ±100. When solving a

rational equation, discard any candidate that makes a denominator zero; for an inequality, use a sign chart instead of multiplying by a variable factor.

LOCK THESE IN

- Degree and leading coefficient decide end behavior.
- Even: ends agree. Odd: ends disagree. Negative leading coefficient flips.
- Odd multiplicity crosses; even multiplicity touches and turns.
- Synthetic division: use c from x − c and 0 placeholders.
- Remainder Theorem: remainder = P(c). Remainder 0 means a factor.
- Domain before canceling. Canceled factor: hole. Surviving factor: vertical asymptote.
- HA: n < m gives y = 0; n = m gives the ratio; n = m + 1 gives a slant line.
- Sign chart: one test value per strip.



f(x) = −(x + 2)(x − 1)²

degree 1 + 2 = 3, odd

leading coefficient −1

odd and negative:

up on the left,

down on the right

odd multiplicity: cross

even multiplicity: touch and turn back

f(0) = −(2)(1) = −2

at most 2 turning points

Two numbers fix the ends; multiplicity fixes each zero. Odd degree with a negative leading coefficient rises on the left and falls on the right. The graph crosses at the single zero and bounces at the double zero.

Polynomial and rational function rules

Rule	Statement	When to use it
End behavior	Even: ends agree. Odd: ends disagree. $a_n < 0$ flips	Sketching any polynomial
Zero and turn limits	≤ n zeros, ≤ n − 1 turning points	Checking a sketch
Multiplicity	Odd: cross. Even: touch and turn	Behavior at each zero
Division identity	$P = D \cdot Q + R$, deg R < deg D	Checking any division
Remainder Theorem	$P(x) \div (x - c)$ leaves $P(c)$	Evaluating or testing a zero
Factor Theorem	$P(c) = 0$ exactly when $(x - c)$ is a factor	Peeling off factors
Rational Zero Theorem	± (factors of a_0)/(factors of a_n)	Listing candidates
Hole vs asymptote	Canceled factor: hole. Surviving: vertical asymptote	Rational graphs
Horizontal asymptote	$n < m$: $y = 0$. $n = m$: $y = a_n/b_m$	Far-end behavior
Slant asymptote	$n = m + 1$: divide, use the quotient	Top degree one more

End behavior at a glance

Degree	Leading coefficient	Left end	Right end
Even	Positive	Up	Up
Even	Negative	Down	Down
Odd	Positive	Down	Up
Odd	Negative	Up	Down

Worked examples

WORKED EXAMPLE 1 · END BEHAVIOR

Describe the ends of $f(x) = 3x^4 - x^3 + 2$ and $g(x) = -x^5 + 4x$.

STEP 1 · F: DEGREE AND SIGN

Degree 4 (even), leading coefficient 3 (positive): **both ends rise**.

STEP 2 · G: DEGREE AND SIGN

Degree 5 (odd), leading coefficient −1 (negative): **rises on the left, falls on the right**.

CHECK F WITH X = 10

30,000 − 1,000 + 2 = 29,002, far above the axis.

WORKED EXAMPLE 2 · ZEROS BY FACTORING

Find all real zeros of $P(x) = x^3 - x^2 - 12x$.

STEP 1 · GCF FIRST

$P(x) = x(x^2 - x - 12)$.

STEP 2 · FACTOR THE TRINOMIAL

Product −12, sum −1: −4 and 3. $P(x) = x(x - 4)(x + 3)$.

STEP 3 · ZEROS

x = 0, x = 4, x = −3, each of multiplicity 1, so the graph crosses at all three.

CHECK X = −3

−27 − 9 + 36 = 0.

WORKED EXAMPLE 3 · READING A FACTORED POLYNOMIAL

Describe $f(x) = (x - 2)^2(x + 1)^3$.

STEP 1 · DEGREE AND LEADING COEFFICIENT

2 + 3 = 5, odd. Leading term x^5, coefficient +1.

STEP 2 · END BEHAVIOR

Odd and positive: falls on the left, rises on the right.

STEP 3 · EACH ZERO

x = 2 has multiplicity 2: touch and turn. x = −1 has multiplicity 3: cross, flattening.

STEP 4 · Y-INTERCEPT

$f(0) = (-2)^2(1)^3 = 4$, so **(0, 4)**.

WORKED EXAMPLE 4 · SYNTHETIC DIVISION AND THE FACTOR THEOREM

Show that $x = 2$ is a zero of $x^3 + 2x^2 - 5x - 6$, then factor it completely.

STEP 1 · SET UP

Coefficients 1, 2, −5, −6 and $c = 2$.

STEP 2 · BRING DOWN, MULTIPLY, ADD

1; 2 + 2 = 4; −5 + 8 = 3; −6 + 6 = 0.

STEP 3 · READ THE ROW

Remainder 0, so $(x - 2)$ is a factor. The quotient is $x^2 + 4x + 3$.

STEP 4 · FACTOR THE QUOTIENT

$x^2 + 4x + 3 = (x + 1)(x + 3)$.

STEP 5 · ANSWER

$(x - 2)(x + 1)(x + 3)$; zeros 2, −1, −3.

WORKED EXAMPLE 5 · A RATIONAL FUNCTION, FEATURE BY FEATURE

Analyze $f(x) = (x^2 - 4)/(x^2 - x - 2)$.

STEP 1 · FACTOR

$[(x - 2)(x + 2)]/[(x - 2)(x + 1)]$.

STEP 2 · DOMAIN FIRST

$x \neq 2$ and $x \neq -1$.

STEP 3 · CANCEL: HOLE

$f(x) = (x + 2)/(x + 1)$ for $x \neq 2$. Hole height $(2 + 2)/(2 + 1) = 4/3$: **hole at (2, 4/3)**.

STEP 4 · ASYMPTOTES

Vertical: **$x = -1$**. Equal degrees, leading coefficients 1 and 1: **$y = 1$**.

STEP 5 · INTERCEPTS

x-intercept (−2, 0). y-intercept $f(0) = 2/1 = 2$, so (0, 2).

TRAPS PEOPLE FALL INTO

- **Reading the leading coefficient before ordering the terms.** Rewrite highest power first; keep its sign.
- **Losing the zero at x = 0.** Pull out the GCF before anything else.

- **Using c = 5 for x + 5.** x + 5 = x − (−5), so c = −5.
- **Skipping placeholders.** $x^4 - 16$ has coefficients 1, 0, 0, 0, −16.
- **Canceling before stating the domain.** The hole disappears from your work, but not from the function.
- **Calling every excluded value an asymptote.** A canceled factor gives a hole.
- **Thinking a graph cannot cross a horizontal asymptote.** It can, in the middle; it cannot cross a vertical one.
- **Multiplying an inequality by a variable factor.** Its sign is unknown; use a sign chart.

Chapter 10 self-test · ten items

1. Which way do the arms of $f(x) = -2x^4 + x$ point?

A. Left down, right up

B. Both ends rise

C. Left up, right down

D. Both ends fall

D. Both ends fall Even degree with a negative leading coefficient: both ends point down.

2. A polynomial has degree 5. What is the most turning points it can have?

A. 4

B. 3

C. 5

D. 6

A. 4 Degree n allows at most n − 1 turning points.

3. What are the real zeros of $x^3 - 16x$?

A. 0, 2 and −2

B. 4 and −4 only

C. 0, 4 and −4

D. 0 and 16

C. 0, 4 and −4 $x(x^2 - 16) = x(x - 4)(x + 4)$. Pulling out x keeps the zero at 0.

4. The graph of $g(x) = (x - 3)^2(x + 5)$ reaches $x = 3$. How does it behave there?

A. It has a vertical asymptote

B. It touches the axis and turns

C. It crosses with a flat spot

D. It crosses the axis

B. It touches the axis and turns Multiplicity 2 is even, so the graph bounces off the axis.

5. What is the degree of $(x + 1)^2(x - 2)^3(x + 4)$?

A. 6

B. 5

C. 8

D. 3

A. 6 Add the multiplicities: 2 + 3 + 1 = 6.

6. Divide $2x^3 - 3x^2 + 4$ by $x + 1$ synthetically. What do you get?

A. $2x^2 - 5x + 5$, remainder 9

B. $2x^2 - 3x + 4$, remainder 0

C. $2x^2 - 5x + 5$, remainder -1

D. $2x^2 - x - 1$, remainder 3

C. $2x^2 - 5x + 5$, remainder -1 Use 2, −3, 0, 4 with c = −1: 2; −5; 5; −1.

7. Use the Remainder Theorem: $P(x) = x^4 - 3x^2 + 2$ divided by $x - 2$ leaves what?

A. −6

B. 6

C. 0

D. 2

B. 6 By the Remainder Theorem, P(2) = 16 − 12 + 2 = 6.

8. Classify the excluded value $x = -1$ of $r(x) = (x + 1)/(x^2 - 1)$.

A. A vertical asymptote

B. Nothing; f(−1) is defined

C. An x-intercept at (−1, 0)

D. A hole; (x + 1) cancels

D. A hole; (x + 1) cancels $x^2 - 1 = (x - 1)(x + 1)$. The (x + 1) cancels, so x = −1 is a hole and x = 1 is the asymptote.

9. Far to the left and right, $(6x^2 - 1)/(3x^2 + 2x)$ levels off at which line?

A. $y = 6$
B. $y = 2$
C. $y = 0$
D. None exists

B. $y = 2$ Equal degrees, so take the ratio of leading coefficients: 6/3 = 2.

10. What is the slant asymptote of $(x^2 + 4x + 1)/(x + 2)$?

A. $y = x + 4$
B. $y = x - 3$
C. $y = x + 2$
D. $y = x + 6$

C. $y = x + 2$ Synthetic division with $c = -2$ on 1, 4, 1 gives 1, 2, remainder −3, so $f(x) = x + 2 - 3/(x + 2)$.

ONE PAGE

Cram sheet

Everything from chapters 9 and 10 in one place. Cover the right column and see if you can rebuild each row.

Quadratics

Name	Formula	When to use it
Quadratic formula	$x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$	Any quadratic in standard form
Discriminant	$b^2 - 4ac$	+: two; 0: one; −: none real
Square root property	$x^2 = k$ gives $\pm\sqrt{k}$	$(x - h)^2 = k$
Completing the square	Add $(b/2)^2$ to both sides	Leading coefficient 1 first
Axis of symmetry	$x = -b/(2a)$	Vertex, max or min
Vertex form	$a(x - h)^2 + k$	Vertex (h, k)
Projectile	$-16t^2 + v_0t + h_0$	Feet and seconds

Polynomials and rational functions

Item	Rule	Note
End behavior	Even agrees, odd disagrees, negative flips	Only the leading term matters far out
Limits	≤ n zeros, ≤ n − 1 turns	Degree n
Multiplicity	Odd crosses, even touches	Sum of multiplicities = degree
Synthetic division	Bring down, multiply by c, add	c from x − c; 0 placeholders
Remainder and Factor	Remainder = P(c); 0 means a factor	Test candidates ± p/q
Domain	Exclude zeros of the original denominator	Before canceling
Hole vs VA	Canceled: hole. Surviving: vertical asymptote	Hole height from the simplified form
Horizontal asymptote	n < m: 0; n = m: ratio; n = m + 1: slant	Check with x = 100

Routines

Task	Steps
Solve a quadratic	Standard form, then factor, square root, complete the square or the formula; check.
Graph a parabola	Direction, axis, vertex, y-intercept, roots, a mirror point.
Graph a rational function	Factor, domain, holes, VAs, intercepts, HA or slant, sign chart.
Applied quadratic	Vertex question or equation question; reject impossible answers; give units.

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