COLLEGE ALGEBRA · STUDY GUIDE

Exam 4 Study Guide

This guide covers chapters 7 and 8. You will simplify, multiply, divide, add and subtract rational expressions, untangle complex fractions and solve rational equations while guarding the excluded values. Then you will simplify and combine radicals, rationalize denominators, switch between radicals and rational exponents, and solve radical equations. Both chapters end with the same lesson: check every answer in the original. Work one chapter at a time, try each worked example with the steps covered, and finish each chapter with its ten self-test items. The cram sheet at the end puts every rule in one place.

WHAT THIS GUIDE COVERS

- Chapter 7 · excluded values, simplifying by factoring, multiplying and dividing, the LCD for adding and subtracting, complex fractions, rational equations and extraneous solutions.
- Chapter 8 · principal and nth roots, simplifying radicals, adding, multiplying and dividing radicals, rationalizing with conjugates, rational exponents and radical equations.

CHAPTER 7

Rational Expressions

Read it in full: **Chapter 7 in the textbook**. Practice it in the lab: **The Point That Goes Missing** and **Solve, Then Test**.

What to know

A **rational expression** is a polynomial divided by a polynomial, P/Q. The denominator can never be zero, because division by zero is undefined. So every rational expression comes with **excluded values**: the numbers that make a denominator zero. Find them first, from the original denominators, by factoring and setting each factor to zero.

Simplify by factoring the top and bottom completely, then dividing out common **factors**: $(a \cdot c)/(b \cdot c) = a/b$. A factor is multiplied; a term is added. You may cancel factors, never terms, so $(x + 4)/4$ does not simplify to x. Opposites differ by −1: $3 - x = -1(x - 3)$, so $(3 - x)/(x - 3) = -1$.

Restrictions survive simplifying. $(x^2 - 9)/(x^2 + x - 12)$ simplifies to $(x + 3)/(x + 4)$, but $x = 3$ is still excluded, and the graph of the original has a hole there. Check every simplification by plugging an allowed number into both forms.

To multiply, factor everything, cancel matching factors, then multiply what is left and leave it factored. Expanding first only creates a long polynomial you must factor again. To divide, flip only the divisor and multiply. Division adds one more restriction: the divisor cannot be zero, so its numerator also gives an excluded value.

Like denominators: combine the numerators and keep the denominator. Unlike denominators need a **least common denominator**. Factor every denominator, list each distinct factor, raise each to the highest power it reaches in any one denominator, and multiply. Rewrite each fraction by multiplying top and bottom by its missing factor. When subtracting, put the second numerator in parentheses so the minus reaches every term: $x - (2x - 6) = -x + 6$.

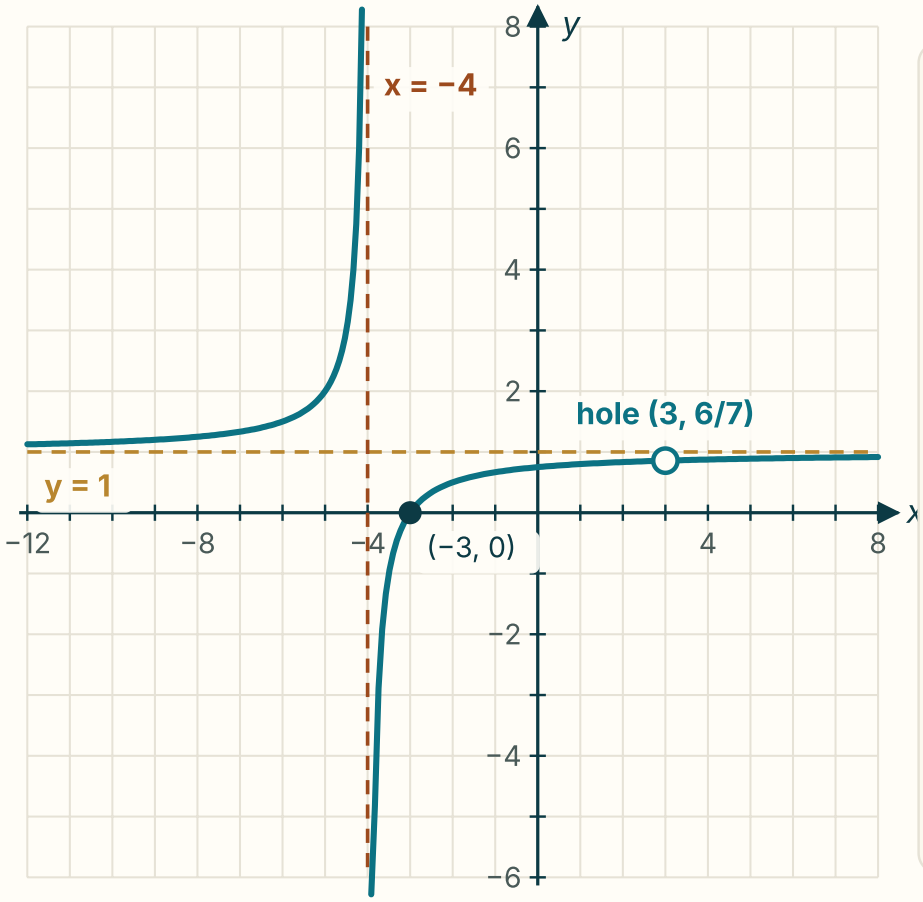
A **complex fraction** has fractions inside it. The main bar means "divided by." Method 1 combines the top into one fraction and the bottom into one fraction, then multiplies by the reciprocal. Method 2 multiplies the whole top and the whole bottom by the LCD of every small fraction inside, which is usually faster. Restrictions hide in three places: inner denominators, the main denominator, and the final answer's denominator.

A **rational equation** has an equals sign, so you may multiply both sides by the LCD and wipe out every denominator. List the excluded values first, clear the denominators, solve, then screen every candidate. A candidate on the excluded list is an **extraneous solution**: it solves the cleared equation but makes the original undefined. If every candidate is extraneous, the answer is "no solution."

Rate problems lead to rational equations. If one pump fills a tank in 6 hours and another in 3, together they do $1/6 + 1/3$ of the job per hour, so $1/6 + 1/3 = 1/t$ gives $t = 2$ hours. A combined time must be shorter than the faster worker alone, which is a quick sanity check.

LOCK THESE IN

- Find excluded values first, from the original denominators.
- Cancel factors, never terms.
- $b - a = -1(a - b)$.
- Multiply: factor, cancel, multiply. Divide: flip the divisor first.
- The divisor's numerator is also an excluded value.
- LCD: every distinct factor at its highest power.
- Parenthesize the second numerator when subtracting.
- Equations: clear with the LCD, then screen for extraneous solutions.



$f(x) = \dfrac{x^2 - 9}{x^2 + x - 12}$

factor:

$$\frac{(x + 3)(x - 3)}{(x + 4)(x - 3)}$$

simplified:

$$\frac{x + 3}{x + 4}$$

Excluded: $x \neq -4, 3$

$x = 3$: the factor canceled, so the graph has a hole.

$x = -4$: still in the denominator, so a vertical asymptote.

Excluded values survive simplifying. The simplified form $(x + 3)/(x + 4)$ looks fine at $x = 3$, but the original is undefined there, so the graph has a hole. A factor that stays in the denominator gives an asymptote instead.

Rules and when to use them

Task	Rule	When to use it
Excluded values	Set each factor of each denominator to 0	Before any other step
Simplify	$(a \cdot c)/(b \cdot c) = a/b$, $b, c \neq 0$	A factor appears top and bottom
Opposites	$(b - a)/(a - b) = -1$	Factors that are reversed
Multiply	$(a/b)(c/d) = ac/bd$	After factoring and canceling
Divide	$(a/b) \div (c/d) = (a/b)(d/c)$, $c \neq 0$	Flip only the divisor
Add or subtract	Rewrite over the LCD, then combine numerators	Unlike denominators
Complex fraction	Multiply top and bottom by the LCD of inner fractions	Several small fractions inside
Rational equation	Multiply every term by the LCD, solve, screen	There is an equals sign
Cross-multiplication	$a/b = c/d$ gives $ad = bc$	Exactly one fraction equals one fraction
Work rate	$1/t_1 + 1/t_2 = 1/t$	Two workers or pumps together

Building an LCD

Denominators	Factored	LCD
$4x$ and $6x^2$	2^2x and $2 \cdot 3x^2$	$12x^2$
$x^2 - 9$ and $x + 3$	$(x - 3)(x + 3)$ and $(x + 3)$	$(x - 3)(x + 3)$
$x^2 + 6x + 9$ and $x^2 - 9$	$(x + 3)^2$ and $(x - 3)(x + 3)$	$(x + 3)^2(x - 3)$

Worked examples

WORKED EXAMPLE 1 · SIMPLIFY AND STATE RESTRICTIONS

Simplify $(x^2 - 4x - 12)/(x^2 - 36)$.

STEP 1 · FACTOR THE DENOMINATOR AND RESTRICT

$x^2 - 36 = (x - 6)(x + 6)$, so $x \neq 6$ and $x \neq -6$.

STEP 2 · FACTOR THE NUMERATOR

$x^2 - 4x - 12 = (x - 6)(x + 2)$, since $(-6)(2) = -12$ and $-6 + 2 = -4$.

STEP 3 · DIVIDE OUT THE COMMON FACTOR

$(x + 2)/(x + 6)$, with $x \neq 6$ and $x \neq -6$.

CHECK AT X = 0

Original: $-12/(-36) = 1/3$. Simplified: $2/6 = 1/3$. They match.

WORKED EXAMPLE 2 · DIVIDING RATIONAL EXPRESSIONS

Divide $(x^2 - 25)/(2x + 8) \div (x - 5)/(x + 4)$.

STEP 1 · FACTOR

$x^2 - 25 = (x - 5)(x + 5)$ and $2x + 8 = 2(x + 4)$.

STEP 2 · RESTRICTIONS

Denominators give $x \neq -4$. The divisor's numerator gives $x \neq 5$.

STEP 3 · FLIP AND MULTIPLY

$[(x - 5)(x + 5)]/[2(x + 4)] \cdot (x + 4)/(x - 5)$.

STEP 4 · CANCEL

$(x - 5)$ and $(x + 4)$ divide out, leaving **$(x + 5)/2$**, with $x \neq -4, 5$.

CHECK AT X = 0

$(-25/8) \div (-5/4) = (-25/8)(-4/5) = 5/2$, and $(0 + 5)/2 = 5/2$.

WORKED EXAMPLE 3 · SUBTRACTING WITH THE LCD

Subtract $4/(x + 2) - (x - 6)/(x^2 - 4)$.

STEP 1 · FACTOR AND RESTRICT

$x^2 - 4 = (x - 2)(x + 2)$, so $x \neq 2$ and $x \neq -2$.

STEP 2 · LCD

$(x - 2)(x + 2)$. The first fraction is missing $(x - 2)$.

STEP 3 · SUBTRACT WITH PARENTHESES

$4(x - 2) - (x - 6) = 4x - 8 - x + 6 = 3x - 2$.

STEP 4 · ANSWER

$(3x - 2)/[(x - 2)(x + 2)]$, with $x \neq \pm 2$.

CHECK AT X = 0

Original: $4/2 - (-6)/(-4) = 2 - 1.5 = 0.5$. Answer: $-2/(-4) = 0.5$.

WORKED EXAMPLE 4 · A COMPLEX FRACTION BY THE LCD METHOD

Simplify $(1/x - 1/4) \div (1/x^2 - 1/16)$.

STEP 1 · LCD OF THE INNER FRACTIONS

The denominators are x, 4, x^2 and 16, so the LCD is $16x^2$.

STEP 2 · MULTIPLY TOP AND BOTTOM

Top: $16x - 4x^2 = 4x(4 - x)$. Bottom: $16 - x^2 = (4 - x)(4 + x)$.

STEP 3 · CANCEL

$4x/(4 + x)$.

STEP 4 · RESTRICTIONS

$x \neq 0$ from the inner fractions, and the main bottom is zero when $x^2 = 16$, so $x \neq 4$ and $x \neq -4$.

CHECK AT X = 1

$(1 - 0.25)/(1 - 0.0625) = 0.75/0.9375 = 0.8$, and $4/5 = 0.8$.

WORKED EXAMPLE 5 · A RATIONAL EQUATION WITH AN EXTRANEOUS CANDIDATE

Solve $2x/(x - 1) = 2/(x - 1) + x$.

STEP 1 · EXCLUDE

$x \neq 1$.

STEP 2 · MULTIPLY EVERY TERM BY (X – 1)

$2x = 2 + x(x - 1) = 2 + x^2 - x$.

STEP 3 · SOLVE

$x^2 - 3x + 2 = 0$, so $(x - 1)(x - 2) = 0$ and $x = 1$ or $x = 2$.

STEP 4 · SCREEN

$x = 1$ is excluded, so it is extraneous. Keep **$x = 2$**.

VERIFY

Left: 4/1 = 4. Right: 2/1 + 2 = 4. Both sides match.

TRAPS PEOPLE FALL INTO

- **Finding restrictions after simplifying.** The canceled factor still makes the original undefined.
- **Canceling terms.** (x + 4)/4 is not x. Only factors divide out.
- **Expanding before canceling.** Factored form is the useful form.
- **Flipping the wrong fraction.** Flip only the divisor, the fraction after ÷.
- **Forgetting the divisor's numerator.** It cannot be zero either, so it adds an excluded value.
- **Using the product of the denominators as the LCD.** Take each factor only to its highest power.
- **Dropping the parentheses when subtracting.** x − (2x − 6) is −x + 6, not −x − 6.
- **Keeping an excluded candidate.** It is extraneous; if all are, the answer is no solution.

Chapter 7 self-test · ten items

1. What are the excluded values of $(x - 1)/(x^2 + 5x + 6)$?

A. $x \neq 1, -2$ and -3

B. $x \neq 2$ and $x \neq 3$

C. $x \neq 1$ only

D. $x \neq -2$ and $x \neq -3$

D. $x \neq -2$ and $x \neq -3$ $x^2 + 5x + 6 = (x + 2)(x + 3)$. Only the denominator creates restrictions.

2. Simplify $(x^2 - 49)/(x^2 - 5x - 14)$.

A. $49/(5x + 14)$

B. $(x + 7)/(x + 2)$

C. $(x - 7)/(x + 2)$

D. $(x + 7)/(x - 2)$

B. $(x + 7)/(x + 2)$ Top: $(x - 7)(x + 7)$. Bottom: $(x - 7)(x + 2)$. Cancel $(x - 7)$, with $x \neq 7, -2$.

3. What does $(5 - x)/(x - 5)$ simplify to?

A. -1, for $x \neq 5$

B. 0, for every x

C. 1, for $x \neq 5$

D. -1, for every x

A. -1, for $x \neq 5$ $5 - x = -1(x - 5)$, so the quotient is -1. The restriction $x \neq 5$ stays.

4. Multiply $(x^2 - 9)/(4x) \cdot (8x^2)/(x + 3)$.

A. $2(x - 3)$

B. $8x(x - 3)$

C. $2x(x - 3)$

D. $2x(x + 3)$

C. $2x(x - 3)$ Factor $x^2 - 9 = (x - 3)(x + 3)$. Cancel $(x + 3)$ and $4x$ from $8x^2$, leaving $2x(x - 3)$, with $x \neq 0, -3$.

5. In $(x + 1)/(x - 2) \div (x - 5)/(x + 3)$, which value is excluded besides 2 and -3?

A. $x = 1$

B. $x = -5$

C. $x = 5$

D. $x = -1$

C. $x = 5$ The divisor $(x - 5)/(x + 3)$ cannot be zero, so its numerator $x - 5$ adds the restriction $x \neq 5$.

6. Which expression is the least common denominator for $1/(x^2 - 9)$ and $1/(x^2 + 6x + 9)$?

A. $(x - 3)(x + 3)$

B. $(x - 3)^2(x + 3)$

C. $(x^2 - 9)(x^2 + 6x + 9)$

D. $(x - 3)(x + 3)^2$

D. $(x - 3)(x + 3)^2$ The factors are $(x - 3)(x + 3)$ and $(x + 3)^2$. Take $(x + 3)$ to its highest power, 2.

7. Write $2/x + 3/(x - 1)$ as a single fraction.

A. $(5x - 2)/[x(x - 1)]$

B. $(5x - 3)/[x(x - 1)]$

C. $(5x + 2)/[x(x - 1)]$

D. $5/(2x - 1)$

A. $(5x - 2)/[x(x - 1)]$ Over $x(x - 1)$: $2(x - 1) + 3x = 5x - 2$. Adding tops and bottoms separately is never allowed.

8. Simplify the complex fraction $(1 - 4/x^2) \div (1 + 2/x)$.

A. $(x + 2)/x$

B. $(x - 2)/x$

C. $(x - 2)/(x + 2)$

D. $x/(x - 2)$

B. $(x - 2)/x$ Multiply top and bottom by x^2: $(x^2 - 4)/(x^2 + 2x) = (x - 2)(x + 2)/[x(x + 2)] = (x - 2)/x$.

9. Find x if 5/x = 7/(x + 4).

A. x = 5/3
B. x = −10
C. x = 4
D. x = 10

D. x = 10 Cross-multiply: 5(x + 4) = 7x, so 5x + 20 = 7x and x = 10. It is not excluded. Check: 5/10 = 0.5 and 7/14 = 0.5.

10. Hose A alone fills a pool in 4 hours; hose B alone needs 12 hours. Running both at once, how many hours does the job take?

A. 3 hours
B. 8 hours
C. 6 hours
D. 16 hours

A. 3 hours 1/4 + 1/12 = 1/t. Multiply by 12t: 3t + t = 12, so t = 3. It is faster than either pump alone.

CHAPTER 8

Radicals and Rational Exponents

Read it in full: **Chapter 8 in the textbook**. Practice it in the lab: **Breaking the Radical Apart** and **When Squaring Invents Answers**.

What to know

√25 asks which non-negative number squares to 25. The symbol returns only the **principal** root: √25 = 5, never −5. Solving $x^2 = 25$ is a different job and gives x = ±5. Because squaring erases a sign, $\sqrt{(x^2)} = |x|$.

The small number in the notch is the **index**, and the number underneath is the **radicand**. No index means 2. An odd index accepts negatives: $\sqrt[3]{(-8)} = -2$. An even index with a negative radicand

has no real value: $\sqrt{(-9)}$ is not real. Know the perfect squares to 225, the cubes to 216, and a few fourth and fifth powers on sight.

A radical is simplified when its radicand has no perfect-power factor matching the index. Use the product property $\sqrt{(ab)} = \sqrt{a} \cdot \sqrt{b}$: $\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$. The index decides what counts as perfect: $\sqrt[3]{54} = 3 \cdot \sqrt[3]{2}$ because 27 is a cube. With variables, divide each exponent by the index: under a square root x^6 comes out as x^3, and x^7 leaves one x inside. Use $|x|$ when an even power of a possibly negative variable comes out.

Only **like radicals**, same index and same radicand, add or subtract, so simplify first to reveal hidden matches: $\sqrt{18} + \sqrt{50} - \sqrt{8} = 3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$. $\sqrt{2} + \sqrt{3}$ does not combine. Radicals never split across a sum: $\sqrt{(9 + 16)} = 5$, not 7. To multiply, multiply outsides with outsides and radicands with radicands, then simplify; use FOIL for two terms, and $\sqrt{a} \cdot \sqrt{a} = a$. To divide, $\sqrt{a}/\sqrt{b} = \sqrt{(a/b)}$ with $b > 0$.

Rationalizing removes a radical from a denominator by multiplying by a disguised 1. For one square root, multiply top and bottom by that root: $3/\sqrt{7} = 3\sqrt{7}/7$. Simplify the radical first if you can. For a cube root, supply the factors that complete a perfect cube: $1/\sqrt[3]{2}$ becomes $\sqrt[3]{4}/2$. For two terms, multiply by the **conjugate**, the same terms with the middle sign flipped, because $(a + b)(a - b) = a^2 - b^2$ wipes out the root.

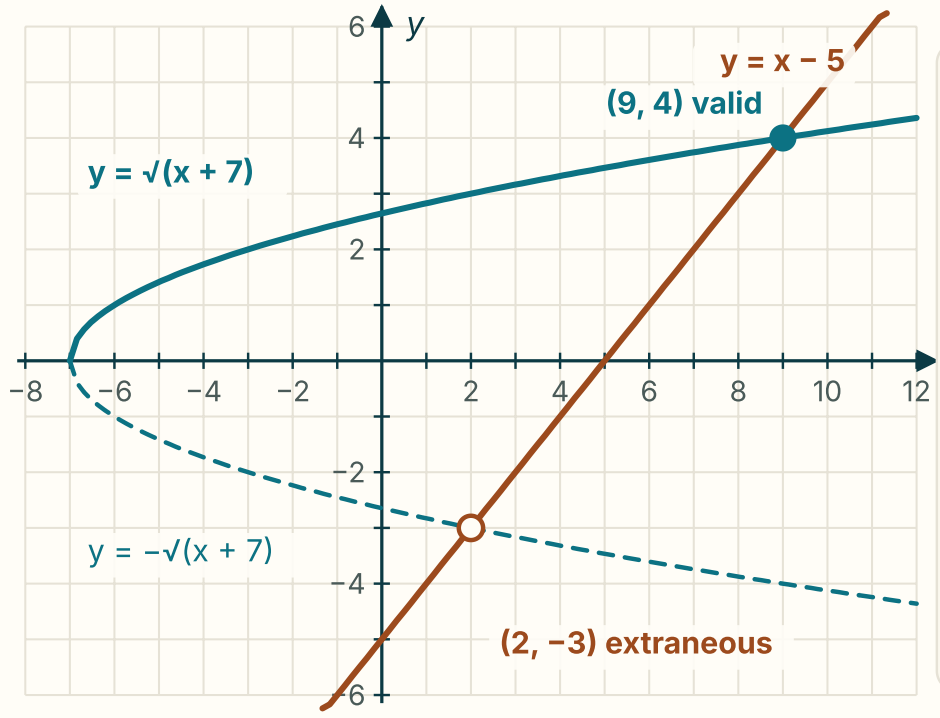
A radical is an exponent: $a^{1/n} = \sqrt[n]{a}$, and $a^{m/n} = (\sqrt[n]{a})^m$. The denominator is the root and the numerator is the power. Take the root first to keep numbers small: $27^{4/3} = 3^4 = 81$. A negative exponent means reciprocal, not a negative answer: $16^{-1/2} = 1/4$. An even denominator needs a non-negative base.

Once radicals are exponents, every exponent rule applies. That is how you combine roots with different indexes: $\sqrt{x} \cdot \sqrt[3]{x} = x^{1/2 + 1/3} = x^{5/6}$. Powers distribute over products: $(16x^8)^{3/4} = 8x^6$.

To solve a **radical equation**, isolate the radical, raise both sides to the index, solve, and check every candidate in the original. Squaring is a one-way street: it turns $3 = -3$ into $9 = 9$, so it can invent **extraneous solutions**. An isolated square root that equals a negative number means no solution. With two radicals, isolate one, square, isolate the other, and square again.

LOCK THESE IN

- $\sqrt{}$ returns the principal root; $x^2 = a$ gives $\pm$.
- Even index and negative radicand: not real. Odd index: fine.
- Simplify with the largest perfect power that matches the index.
- Only like radicals combine; $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$.
- One-term denominator: multiply by the radical. Two terms: the conjugate.
- $a^{m/n} = (\sqrt[n]{a})^m$: root in the denominator, power on top.
- Negative exponent means reciprocal.
- Radical equations: isolate, raise, solve, check every candidate.



Solve $\sqrt{(x + 7)} = x - 5$

square both sides:

$x + 7 = x^2 - 10x + 25$

$0 = (x - 2)(x - 9)$

check x = 9: 4 = 4

check x = 2: 3 ≠ −3

Squaring also solves the dashed mirror equation, so x = 2 rides along.

Squaring cannot tell a root from its mirror. Both branches square to the same equation, so the algebra returns x = 2 and x = 9. Only the check in the original keeps x = 9 and rejects x = 2.

Radical rules and when to use them

Rule	Formula	When to use it
Principal root	$\sqrt{(x^2)} = \lvert x \rvert$	Taking a square root of a square
Product property	$\sqrt{(ab)} = \sqrt{a} \cdot \sqrt{b}$, $a, b \geq 0$	Simplifying and multiplying
Quotient property	$\sqrt{a}/\sqrt{b} = \sqrt{(a/b)}$, $b > 0$	Dividing radicals
Like radicals	$m\sqrt{a} + n\sqrt{a} = (m + n)\sqrt{a}$	Same index and radicand only
Rationalize, one term	$a/\sqrt{b} = a\sqrt{b}/b$	A single square root below
Rationalize, two terms	Multiply by the conjugate; $(a + b)(a - b) = a^2 - b^2$	A sum or difference below
Rational exponent	$a^{1/n} = \sqrt[n]{a}$; $a^{m/n} = (\sqrt[n]{a})^m$	Converting forms
Negative rational exponent	$a^{-m/n} = 1/a^{m/n}$	Clearing a negative exponent
Different indexes	Convert to exponents, add them	$\sqrt{x} \cdot \sqrt[3]{x} = x^{5/6}$
Radical equation	Isolate, raise to the index, solve, check	Variable under a radical

Worked examples

WORKED EXAMPLE 1 · SIMPLIFYING THREE RADICALS

Simplify $\sqrt{180}$, $\sqrt[3]{(-128)}$ and $\sqrt{(48x^5)}$ for $x \geq 0$.

STEP 1 · LARGEST PERFECT SQUARE IN 180

$180 = 36 \cdot 5$, so $\sqrt{180} = \mathbf{6\sqrt{5}}$.

STEP 2 · PERFECT CUBE IN −128

$-128 = -64 \cdot 2$, so $\sqrt[3]{(-128)} = \mathbf{-4 \cdot \sqrt[3]{2}}$. An odd index allows the negative.

STEP 3 · SPLIT THE VARIABLE RADICAND

$48x^5 = 16x^4 \cdot 3x$.

STEP 4 · TAKE THE PERFECT PART OUT

$\sqrt{(16x^4)} = 4x^2$, so the result is **$4x^2\sqrt{(3x)}$**.

CHECK ONE

$\sqrt{180} \approx 13.416$ and $6\sqrt{5} \approx 6 \times 2.236 = 13.416$.

WORKED EXAMPLE 2 · COMBINING AFTER SIMPLIFYING

Simplify $2\sqrt{45} - \sqrt{20} + \sqrt{80}$.

STEP 1 · SIMPLIFY EACH

$\sqrt{45} = 3\sqrt{5}$, $\sqrt{20} = 2\sqrt{5}$, $\sqrt{80} = 4\sqrt{5}$.

STEP 2 · REWRITE

$2(3\sqrt{5}) - 2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5} - 2\sqrt{5} + 4\sqrt{5}$.

STEP 3 · COMBINE LIKE RADICALS

$6 - 2 + 4 = 8$, so **$8\sqrt{5}$**.

CHECK

$13.416 - 4.472 + 8.944 = 17.889$, and $8\sqrt{5} \approx 17.889$.

WORKED EXAMPLE 3 · FOIL WITH RADICALS

Multiply $(2\sqrt{3} - 1)(\sqrt{3} + 4)$.

STEP 1 · FIRST

$2\sqrt{3} \cdot \sqrt{3} = 2 \cdot 3 = 6$.

STEP 2 · OUTER AND INNER

$2\sqrt{3} \cdot 4 = 8\sqrt{3}$ and $-1 \cdot \sqrt{3} = -\sqrt{3}$.

STEP 3 · LAST

$-1 \cdot 4 = -4$.

STEP 4 · COMBINE

$(6 - 4) + (8 - 1)\sqrt{3} =$ **$2 + 7\sqrt{3}$**.

CHECK

$(3.4641 - 1)(1.7321 + 4) = 2.4641 \times 5.7321 = 14.124$, and $2 + 7(1.7321) = 14.124$.

WORKED EXAMPLE 4 · RATIONALIZING WITH A CONJUGATE

Rationalize $10/(4 - \sqrt{6})$.

STEP 1 · CONJUGATE

Flip only the middle sign: $4 + \sqrt{6}$.

STEP 2 · DENOMINATOR

$(4 - \sqrt{6})(4 + \sqrt{6}) = 16 - 6 = 10$.

STEP 3 · NUMERATOR

$10(4 + \sqrt{6}) = 40 + 10\sqrt{6}$.

STEP 4 · REDUCE

$(40 + 10\sqrt{6})/10 = \mathbf{4 + \sqrt{6}}$.

CHECK

$10/(4 - 2.4495) = 10/1.5505 = 6.4495$, and $4 + 2.4495 = 6.4495$.

WORKED EXAMPLE 5 · RATIONAL EXPONENTS

Evaluate $16^{3/4}$ and $125^{-2/3}$, and simplify $x^{1/2} \cdot x^{1/4}$.

STEP 1 · ROOT FIRST

$\sqrt[4]{16} = 2$, then $2^3 = \mathbf{8}$.

STEP 2 · RECIPROCAL, THEN ROOT, THEN POWER

$125^{-2/3} = 1/125^{2/3}$. $\sqrt[3]{125} = 5$ and $5^2 = 25$, so the value is **1/25**.

STEP 3 · ADD EXPONENTS

$1/2 + 1/4 = 3/4$, so $x^{1/2} \cdot x^{1/4} = \mathbf{x^{3/4}} = \sqrt[4]{(x^3)}$.

WORKED EXAMPLE 6 · A RADICAL EQUATION WITH AN IMPOSTOR

Solve $\sqrt{(2x + 7)} = x + 2$.

STEP 1 · SQUARE BOTH SIDES

$2x + 7 = x^2 + 4x + 4$.

STEP 2 · COLLECT

$0 = x^2 + 2x - 3 = (x + 3)(x - 1)$, so $x = -3$ or $x = 1$.

STEP 3 · CHECK X = 1

$\sqrt{9} = 3$ and $1 + 2 = 3$. Valid.

STEP 4 · CHECK X = −3

$\sqrt{1} = 1$ but $-3 + 2 = -1$. Extraneous.

STEP 5 · ANSWER

x = 1 only. The right side $x + 2$ had to be at least 0, which rules out -3 from the start.

TRAPS PEOPLE FALL INTO

- **Writing √25 = ±5.** The symbol returns 5. The ± comes from solving $x^2 = 25$.
- **Taking an even root of a negative.** √(−9) and $\sqrt[4]{(-16)}$ are not real numbers.
- **Splitting a radical across a sum.** √(9 + 16) = 5, not 3 + 4 = 7.
- **Adding unlike radicals.** √2 + √3 is not √5.
- **Writing the conjugate with the terms swapped.** The conjugate of 3 + √5 is 3 − √5: same order, flipped middle sign.
- **Treating a negative exponent as a negative answer.** $16^{-1/2} = 1/4$, a positive number.
- **Squaring before isolating the radical.** Get the radical alone first, or the square makes a mess.
- **Skipping the check.** Squaring can invent solutions; only the original equation catches them.

Chapter 8 self-test · ten items

1. Only one of these radicals has a real value. Which one?

A. √(−27)
B. √(−4)
C. $\sqrt[4]{(-16)}$
D. $\sqrt[3]{(-27)}$

D. $\sqrt[3]{(-27)}$ An odd index accepts a negative radicand: $\sqrt[3]{(-27)} = -3$. Even indexes with negative radicands are not real.

2. Simplify √98.

A. 7√2
B. 14√7
C. 2√7
D. 49√2

A. 7√2 98 = 49 · 2, and √49 = 7.

3. Simplify $\sqrt[3]{250}$.

A. $25 \cdot \sqrt[3]{10}$

B. $5 \cdot \sqrt[3]{2}$

C. $10 \cdot \sqrt[3]{5}$

D. $2 \cdot \sqrt[3]{5}$

B. $5 \cdot \sqrt[3]{2}$ $250 = 125 \cdot 2$, and $125 = 5^3$.

4. What is $\sqrt{12} + \sqrt{75}$?

A. $7\sqrt{6}$

B. $\sqrt{87}$

C. $7\sqrt{3}$

D. $3\sqrt{7}$

C. $7\sqrt{3}$ $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{75} = 5\sqrt{3}$. Like radicals add: $7\sqrt{3}$.

5. Multiply $\sqrt{5} \cdot \sqrt{15}$ and simplify.

A. $15\sqrt{5}$

B. $3\sqrt{5}$

C. $\sqrt{20}$

D. $5\sqrt{3}$

D. $5\sqrt{3}$ $\sqrt{75} = \sqrt{(25 \cdot 3)} = 5\sqrt{3}$.

6. What is $(\sqrt{2} + 3)(\sqrt{2} - 3)$?

A. 11

B. −1

C. −7

D. 7

C. −7 Conjugates: $(\sqrt{2})^2 - 3^2 = 2 - 9 = -7$.

7. Rewrite $10/\sqrt{5}$ with a rational denominator.

A. $2\sqrt{5}$

B. $5\sqrt{2}$

C. $\sqrt{5}/2$

D. $10\sqrt{5}$

A. $2\sqrt{5}$ Multiply top and bottom by $\sqrt{5}$: $10\sqrt{5}/5 = 2\sqrt{5}$.

8. Rewrite $6/(\sqrt{7} + 1)$ with a rational denominator.

A. $(\sqrt{7} - 1)/6$

B. $\sqrt{7} - 1$

C. $\sqrt{7} + 1$

D. $6\sqrt{7} - 6$

B. $\sqrt{7} - 1$ Multiply by the conjugate $\sqrt{7} - 1$: the bottom is $7 - 1 = 6$, and $6(\sqrt{7} - 1)/6 = \sqrt{7} - 1$.

9. What is the value of $32^{3/5}$?

A. 1/8

B. 8

C. 2

D. 19.2

B. 8 $\sqrt[5]{32} = 2$, then $2^3 = 8$. The root comes from the denominator, the power from the numerator.

10. Find every solution of $\sqrt{(x + 6)} = x$.

A. $x = -2$ only

B. No real value

C. $x = 3$ and $x = -2$

D. $x = 3$ only

D. x = 3 only Squaring gives $x^2 - x - 6 = 0$, so $x = 3$ or $x = -2$. $\sqrt{4} = 2$, not -2, so -2 is extraneous.

ONE PAGE

Cram sheet

Everything from chapters 7 and 8 in one place. Cover the right column and see if you can rebuild each row.

Rational expressions

Item	Rule	Note
Excluded values	Zeros of every original denominator	Listed first; they survive canceling
Simplify	Factor, cancel factors	Never cancel terms
Opposites	$(b - a)/(a - b) = -1$	Factor out -1
Multiply and divide	Factor, flip the divisor, cancel, multiply	Divisor numerator is excluded too
LCD	Each distinct factor at its highest power	Not the plain product
Subtract	Parenthesize the second numerator	Distribute the minus
Complex fraction	Multiply top and bottom by the inner LCD	Three places to find restrictions
Equation	Clear with the LCD, solve, screen	All extraneous means no solution

Radicals and exponents

Name	Formula	When to use it
Principal root	$\sqrt{(x^2)} = \lvert x \rvert$	Square root of a square
Product property	$\sqrt{(ab)} = \sqrt{a}\sqrt{b}$	Simplify by perfect powers
Quotient property	$\sqrt{a}/\sqrt{b} = \sqrt{(a/b)}$	Divide radicals
Conjugate	$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$	Two-term denominators
Rational exponent	$a^{m/n} = (\sqrt[n]{a})^m$	Root first
Negative exponent	$a^{-m/n} = 1/a^{m/n}$	Reciprocal, still positive
Work rate	$1/t_1 + 1/t_2 = 1/t$	Jobs done together

Checks that catch errors

Situation	The check
Any simplification	Plug an allowed number into both forms.
Rational equation	Compare each candidate with the excluded list.
Radical equation	Substitute every candidate in the original equation.
Radical arithmetic	Compare decimal values of the start and the answer.

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