COLLEGE ALGEBRA · STUDY GUIDE

Exam 3 Study Guide

This guide covers chapters 5 and 6. You will solve systems of equations three ways, reduce a three-variable system, set up mixture and current problems and shade systems of inequalities. Then you will name, add, subtract and multiply polynomials and learn to factor them in a fixed order. Work one chapter at a time. Read the "what to know" paragraphs, copy the tables out by hand, then try each worked example with the steps covered. Finish each chapter with its ten self-test items. The cram sheet at the end puts every pattern in one place.

WHAT THIS GUIDE COVERS

- Chapter 5 · what a solution is, graphing and the three outcomes, substitution, elimination, three-variable systems, mixture and rate applications, and systems of linear inequalities.
- Chapter 6 · polynomial vocabulary, adding and subtracting, multiplying and special products, the GCF and grouping, trinomials and the AC method, and the special factoring patterns.

CHAPTER 5

Systems of Equations and Inequalities

Read it in full: **Chapter 5 in the textbook**. Practice it in the lab: **Where Two Lines Meet** and **Making a Variable Disappear**.

What to know

A **system of equations** is two or more equations that must be true at the same time. A solution in two variables is an **ordered pair** that makes every equation true, so always check it in every original equation. Checking one equation and stopping is the most common way to lose points here.

Each line is the picture of the pairs that satisfy its equation, so a pair that satisfies both must lie on both lines: the **intersection**. Graphing builds understanding, but it reads badly when the answer is

not a whole number. Use algebra for exact values and the graph as a sanity check.

Two lines can relate in only three ways. Different slopes cross once: one solution, a **consistent, independent** system. Same slope with different intercepts are parallel: no solution, an **inconsistent** system. Same slope and same intercept are one line: infinitely many solutions, a **consistent, dependent** system. Compare slopes in $y = mx + b$ form before doing any work.

Substitution trades a variable for an equal expression. Isolate one variable, substitute into the other equation, solve the one-variable equation, back-substitute, and check. It is fastest when a variable is already alone or has a coefficient of 1 or -1. Substituting back into the same equation gives a useless true statement such as $6 = 6$. Substitution also handles nonlinear systems, where you must state excluded values first and test every candidate.

Elimination adds the equations so one variable cancels. If $A = B$ and $C = D$, then $A + C = B + D$. When the coefficients are not opposites, multiply a whole equation, every term including the constant, by a number chosen from the least common multiple. Multiplying an equation by a nonzero number does not change its solutions. Elimination is best for $Ax + By = C$ systems and for three or more variables.

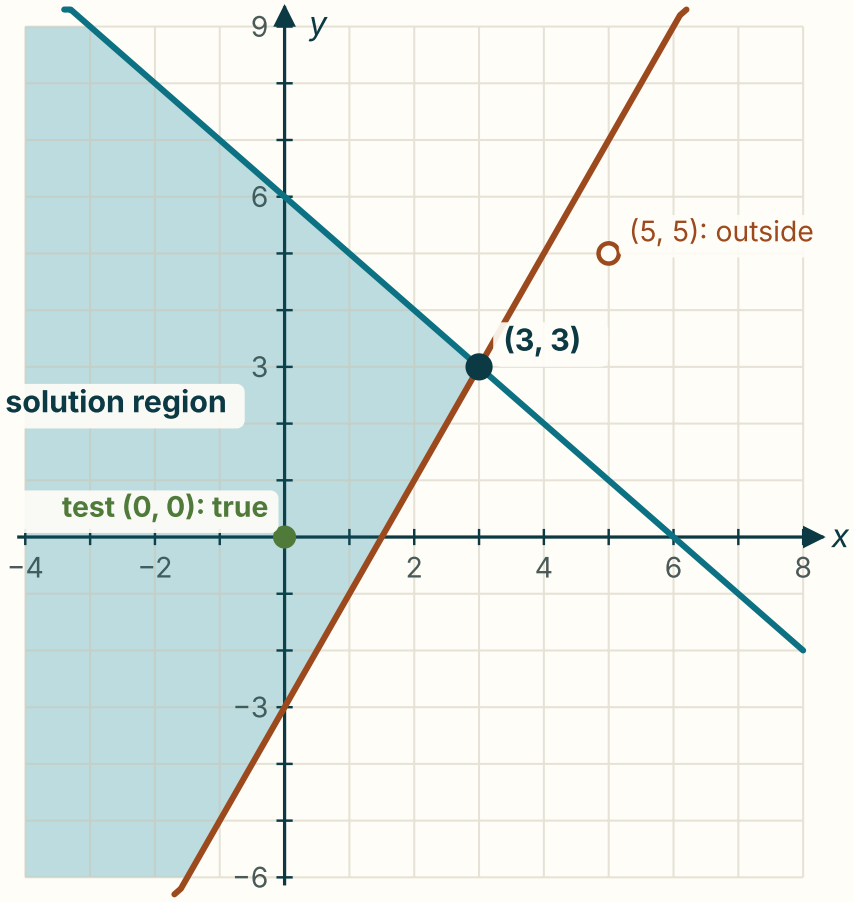
Both methods end the same way in the special cases. If every variable cancels and the statement is false, such as $0 = -9$, there is no solution. If it is true, such as $0 = 0$, there are infinitely many, and you report them as all pairs on one of the lines.

A system in three variables has an **ordered triple** as its solution, and each equation is a plane. Eliminate the **same** variable from two different pairs of equations to get a 2×2 system, solve it, then back-substitute. Use every equation at least once, line up the columns, and check the triple in all three originals.

In applications, name each unknown with a unit and write one equation per independent fact. A mixture problem needs an amount equation and a substance equation: $p_1x + p_2y = p_{mix}$(total). A current or wind problem uses rate with the flow $= b + c$ and against the flow $= b - c$. A **system of inequalities** is solved by shading: solid boundary for $\leq$ and $\geq$, dashed for $<$ and $>$, a test point to choose each side, and the answer is the overlap, which can be empty.

LOCK THESE IN

- A solution must satisfy every equation. Check them all.
- Different slopes: one solution. Parallel: none. Same line: infinitely many.
- Substitution: isolate, substitute into the other equation, solve, back-substitute, check.
- Elimination: multiply whole equations so one variable has opposite coefficients, then add.
- False ending (0 = 7): no solution. True ending (0 = 0): infinitely many.
- 3 × 3: eliminate the same variable twice, using every equation.
- Mixture: amount equation plus substance equation. Current: b + c and b − c.
- Inequalities: solid for ≤ ≥, dashed for < >, test a point, keep the overlap.



y ≤ −x + 6
solid; shade below

y ≥ 2x − 3
solid; shade above

As equations, the lines cross once at (3, 3): one solution, slopes −1 and 2 differ.

As inequalities, the answer is the overlap, a wedge with its point at (3, 3). Solid edges belong to the region.

One crossing point for equations; one shaded overlap for inequalities. Different slopes guarantee exactly one intersection. For the inequalities, a test point picks each side, and the answer is only where both shadings agree.

Methods and models, with when to use each

Tool	Form or rule	When to use it
Graphing	Read the intersection of the two lines	Understanding; checking an answer
Substitution	Isolate a variable, substitute into the other equation	A variable is alone or has coefficient ±1; nonlinear systems
Elimination	Multiply to make opposite coefficients, then add	Ax + By = C with coefficients 2 or more; three variables
Classify first	Compare m and b in y = mx + b	Before any solving
3 × 3 reduction	Eliminate one variable from two pairs	Three equations, three unknowns
Mixture	$x + y = \text{total}$; $p_1x + p_2y = p_{mix} \cdot \text{total}$	Combining solutions of different strength
Current or wind	$b + c = d/t_1$; $b - c = d/t_2$	Travel with and against a flow
Break-even	Cost = fixed + (per unit)x; Revenue = (price)x; set C = R	Business models
Inequality boundary	Solid for ≤ ≥; dashed for < >	Graphing each inequality
Test point	Use (0, 0) if the line misses it	Choosing the side to shade

Worked examples

WORKED EXAMPLE 1 · SUBSTITUTION

Solve $y = 4x - 9$ and $3x + 2y = 15$.

STEP 1 · THE FIRST EQUATION IS ALREADY SOLVED FOR Y

Substitute $4x - 9$ for y in the second: $3x + 2(4x - 9) = 15$.

STEP 2 · DISTRIBUTE AND COMBINE

$3x + 8x - 18 = 15$, so $11x - 18 = 15$.

STEP 3 · SOLVE

$11x = 33$, so $x = 3$.

STEP 4 · BACK-SUBSTITUTE

$y = 4(3) - 9 = 3$.

STEP 5 · CHECK BOTH ORIGINALS

$y = 4(3) - 9 = 3$ is true, and $3(3) + 2(3) = 9 + 6 = 15$ is true. The solution is **(3, 3)**.

WORKED EXAMPLE 2 · ELIMINATION AFTER MULTIPLYING

Solve $4x + 3y = 5$ and $3x - 2y = 8$.

STEP 1 · CHOOSE A TARGET

Eliminate y. The coefficients 3 and −2 have least common multiple 6.

STEP 2 · MULTIPLY WHOLE EQUATIONS

First times 2: $8x + 6y = 10$. Second times 3: $9x - 6y = 24$.

STEP 3 · ADD

$17x = 34$, so $x = 2$.

STEP 4 · BACK-SUBSTITUTE IN AN ORIGINAL

$4(2) + 3y = 5$, so $3y = -3$ and $y = -1$.

STEP 5 · CHECK

$4(2) + 3(-1) = 5$ and $3(2) - 2(-1) = 6 + 2 = 8$. The solution is **(2, −1)**.

WORKED EXAMPLE 3 · READING THE SPECIAL ENDINGS

Classify $6x - 4y = 10$ with $3x - 2y = 5$, then $6x - 4y = 10$ with $3x - 2y = 4$.

STEP 1 · FIRST PAIR

Multiply the second by −2: $-6x + 4y = -10$. Add to the first: $0 = 0$, which is true. **Infinitely many solutions**: the equations are one line.

STEP 2 · SECOND PAIR

Multiply the second by −2: $-6x + 4y = -8$. Add: $0 = 2$, which is false. **No solution**: the lines are parallel.

CONFIRM WITH SLOPES

All four equations have slope 3/2. The intercepts are −5/2 and −5/2 in the first pair, but −5/2 and −2 in the second.

WORKED EXAMPLE 4 · THREE EQUATIONS, THREE UNKNOWNS

Solve $x + y + z = 4$, $2x + y - z = 0$ and $x - y + 2z = 9$.

STEP 1 · PICK Y TO ELIMINATE

The y-coefficients are 1, 1 and −1, so adding the third equation to each of the others removes y.

STEP 2 · SECOND PLUS THIRD

3x + z = 9. Call it A.

STEP 3 · FIRST PLUS THIRD

2x + 3z = 13. Call it B. Every equation has now been used.

STEP 4 · SOLVE A AND B

From A, z = 9 − 3x. Then 2x + 3(9 − 3x) = 13, so −7x = −14 and x = 2. Then z = 3.

STEP 5 · BACK-SUBSTITUTE

In the first: 2 + y + 3 = 4, so y = −1. The triple is **(2, −1, 3)**.

CHECK ALL THREE

2 − 1 + 3 = 4; 4 − 1 − 3 = 0; 2 + 1 + 6 = 9. All true.

WORKED EXAMPLE 5 · A MIXTURE PROBLEM

How many liters of a 15% solution and a 40% solution make 50 L of a 25% solution?

STEP 1 · NAME THE UNKNOWNS

x = liters of 15%, y = liters of 40%, both at least 0.

STEP 2 · AMOUNT EQUATION

x + y = 50.

STEP 3 · SUBSTANCE EQUATION

0.15x + 0.40y = 0.25(50) = 12.5. Times 100: 15x + 40y = 1,250.

STEP 4 · SUBSTITUTE

x = 50 − y: 15(50 − y) + 40y = 1,250, so 750 + 25y = 1,250 and y = 20. Then x = 30.

STEP 5 · CHECK AGAINST THE STORY

0.15(30) + 0.40(20) = 4.5 + 8 = 12.5 L, and 12.5 ÷ 50 = 25%. Use **30 L of 15% and 20 L of 40%**. More of the weaker one makes sense, since 25% is closer to 15%.

TRAPS PEOPLE FALL INTO

- **Checking only one equation.** A pair must satisfy every equation in the system.
- **Substituting back into the same equation.** You get a true statement such as 6 = 6 and learn nothing.
- **Multiplying only the variable terms.** 2x + 5y = 9 times 3 is 6x + 15y = 27, not 6x + 15y = 9.
- **Writing "x = 0" for a false ending.** A false ending means no solution at all.
- **Eliminating different variables from the two pairs.** In a 3 × 3, cancel the same variable both times.

- **Tracking the liquid instead of the substance.** The second mixture equation counts salt or acid, not total volume.
- **Forgetting to flip when dividing by a negative.** $2x - 3y \geq 6$ becomes $y \leq (2/3)x - 2$.
- **Testing a point that lies on the boundary.** Choose a test point off the line, such as (0, 0) when the line misses it.

Chapter 5 self-test · ten items

1. Check the pair (2, −1) in the system x + y = 1, 3x − y = 7. What do you find?

A. No; it fails the second equation

B. No; it fails both of the equations

C. Yes; it makes both equations true

D. No; it fails the first equation

C. Yes; it makes both equations true $2 + (-1) = 1$, and $3(2) - (-1) = 7$. Both are true, so it is a solution.

2. How many solutions does the system y = −3x + 2, y = −3x − 5 have?

A. Exactly one; it is independent

B. Infinitely many; it is dependent

C. Exactly two; the lines cross twice

D. None; the system is inconsistent

D. None; the system is inconsistent Equal slopes of −3 with different intercepts mean parallel lines that never meet.

3. How would you classify $4x - 6y = 8$ and $2x - 3y = 4$?

A. Dependent: infinitely many solutions
B. Independent: exactly two solutions
C. Inconsistent: no solution at all
D. Independent: exactly one solution

A. Dependent: infinitely many solutions The first equation is the second times 2, so both describe one line.

4. Use substitution: $y = 2x + 1$ and $5x - y = 8$. What is the solution?

A. (3, 5)
B. (3, 7)
C. (7, 3)
D. (−3, −5)

B. (3, 7) $5x - (2x + 1) = 8$, so $3x = 9$ and $x = 3$. Then $y = 7$. Check: $15 - 7 = 8$.

5. Use elimination: $3x + 2y = 16$ and $5x - 2y = 0$. What is the solution?

A. (2, −5)
B. (4, 2)
C. (5, 2)
D. (2, 5)

D. (2, 5) Adding cancels y: $8x = 16$, $x = 2$. Then $6 + 2y = 16$, so $y = 5$.

6. While solving a system, every variable cancels and you get 0 = 0. What does that mean?

A. Infinitely many solutions on one line
B. The single solution is x = 0, y = 0
C. No solution, because the lines are parallel
D. An arithmetic error was surely made

A. Infinitely many solutions on one line A true statement with no variables means the equations describe the same line.

7. Which method is usually fastest for 4x + 7y = 3 and 6x − 5y = 2?

A. Graphing, for an exact answer
B. Substitution, isolating x first
C. Elimination; no coefficient is 1
D. Substitution, since y is alone

C. Elimination; no coefficient is 1 Every coefficient is 2 or more, so substitution would force fractions at once. Graphing rarely gives exact values.

8. Solve the triangular system z = 4, y + z = 3, x + y + z = 5.

A. (4, −1, 2)
B. (2, −1, 4)
C. (2, 1, 4)
D. (−2, −1, 4)

B. (2, −1, 4) Work from the bottom: z = 4, so y = 3 − 4 = −1, and x = 5 − (−1) − 4 = 2.

9. A boat goes 30 miles downstream in 2 hours and back upstream in 3 hours. What is the speed of the current?

A. 12.5 mi/h
B. 10.0 mi/h
C. 2.5 mi/h
D. 5.0 mi/h

C. 2.5 mi/h $b + c = 15$ and $b - c = 10$. Adding gives $2b = 25$, so $b = 12.5$ and $c = 2.5$.

10. How do you graph $y > -2x + 4$?

A. Dashed line; shade above, away from (0, 0)
B. Solid line; shade above, away from (0, 0)
C. Dashed line; shade below, toward (0, 0)
D. Solid line; shade below, toward (0, 0)

A. Dashed line; shade above, away from (0, 0) The strict $>$ means a dashed boundary. Testing (0, 0) gives $0 > 4$, false, so shade the side without the origin.

CHAPTER 6

Polynomials and Factoring

Read it in full: **Chapter 6 in the textbook**. Practice it in the lab: **Build the Rectangle** and **The Factor-Pair Hunt**.

What to know

A **polynomial** is a sum of terms in which every exponent on a variable is a whole number and no variable sits in a denominator, under a radical or in an exponent. So $4x^3 - x + 9$ is a polynomial, but $3/x$, $\sqrt{x}$ and 2^x are not. The sign in front of a term belongs to it: in $4x^3 - x + 9$ the coefficient of x is -1.

The **degree** of a term is its exponent, or the sum of exponents when there are several variables. The degree of the polynomial is the largest of these. **Standard form** lists terms from highest degree to lowest, and the first coefficient is the **leading coefficient**. Polynomials are named by terms (monomial, binomial, trinomial) and by degree (constant, linear, quadratic, cubic, quartic).

Add and subtract by combining **like terms**: same variable, same power. Exponents never change when you add: $3x^2 + 4x^2 = 7x^2$. A minus sign in front of a group is −1 times the group, so it flips every sign inside.

To multiply, send every term of one factor through every term of the other, then combine like terms. Here the exponents do add. FOIL (First, Outer, Inner, Last) is that idea for two binomials only. Three special products are worth knowing cold: $(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$, and $(a + b)(a - b) = a^2 - b^2$. Squaring a binomial always makes the 2ab middle term.

Factoring writes a polynomial as a product, and you check it by multiplying back out. Always pull out the **greatest common factor** first: the GCF of the coefficients times each shared variable at its lowest power. A negative leading coefficient is usually pulled out too, which flips every sign inside. Four terms suggest **grouping**: factor each pair, then pull out the matching binomial, factoring out −1 if the parentheses differ only by sign.

For $x^2 + bx + c$, find two numbers with product c and sum b. A negative c means opposite signs, and the larger number carries the sign of b. For $ax^2 + bx + c$ with $a > 1$, use the **AC method**: find two numbers with product $a \cdot c$ and sum b, split the middle term, then group. Walk through the factor pairs in order so you never miss one. A trinomial with no such pair is **prime**, and prime is a real answer.

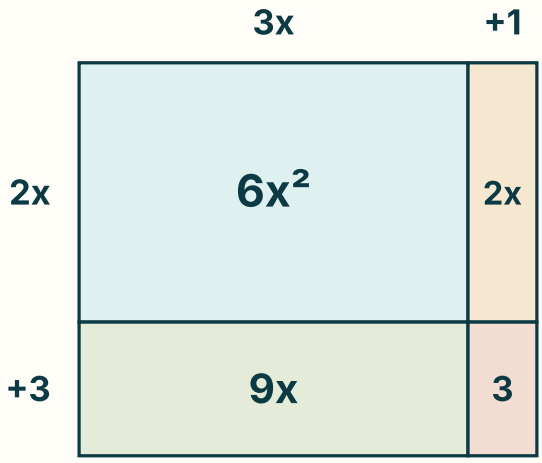
The special patterns run the special products backward. A **difference of squares** $a^2 - b^2 = (a + b)(a - b)$. A sum of squares such as $x^2 + 9$ is prime over the reals. A **perfect square trinomial** needs the middle term to equal exactly 2ab. Cubes follow SOAP: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$: same sign, opposite sign, always positive.

"Factor completely" means keep going until nothing breaks down further. Pull the GCF, then look again: $18x^3 - 50x$ becomes $2x(3x + 5)(3x - 5)$, and $x^4 - 81$ becomes $(x^2 + 9)(x + 3)(x - 3)$. When

you cancel a factor in a fraction, the excluded value stays: $(x^2 - 9)/(x - 3) = x + 3$ only for $x \neq 3$.

LOCK THESE IN

- Polynomial: whole-number exponents; no variable in a denominator, radical or exponent.
- Degree is the largest exponent; standard form runs highest to lowest.
- Subtracting a group flips every sign inside it.
- Multiply: every term times every term; exponents add.
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$, never $a^2 + b^2$.
- GCF first, every time. Four terms: try grouping.
- AC method: product $a \cdot c$, sum b, split, group.
- $a^2 - b^2$ factors; $a^2 + b^2$ is prime; cubes use SOAP.



Area = (2x + 3)(3x + 1)

= 6x² + 9x + 2x + 3

= 6x² + 11x + 3

AC method for 6x² + 11x + 3

1. $a \cdot c = 6 \cdot 3 = 18$
2. product 18, sum 11:

   $1 + 18 = 19$, $2 + 9 = 11$
3. split: $6x^2 + 9x + 2x + 3$
4. group: $3x(2x + 3) + 1(2x + 3)$
5. **factor: $(2x + 3)(3x + 1)$**
6. check by FOIL:

   $6x^2 + 2x + 9x + 3$

The two middle boxes are 9x and 2x.

Factoring is the rectangle run backward. The corner boxes are fixed by a and c, and the two middle boxes must multiply to $a \cdot c = 18$ and add to $b = 11$. That is the whole AC method.

Factoring patterns and when to use them

Pattern	Formula	When to use it
GCF	$ab + ac = a(b + c)$	Always first
Grouping	$ax + ay + bx + by = (x + y)(a + b)$	Four terms
Simple trinomial	$x^2 + bx + c = (x + p)(x + q)$, $pq = c$, $p + q = b$	Leading coefficient 1
AC method	Split b into two numbers with product ac	$ax^2 + bx + c$ with $a > 1$
Difference of squares	$a^2 - b^2 = (a + b)(a - b)$	Two perfect squares with a minus
Sum of squares	$a^2 + b^2$ is prime over the reals	Do not try to factor it
Perfect square	$a^2 \pm 2ab + b^2 = (a \pm b)^2$	Ends are squares and the middle is 2ab
Sum of cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	Two cubes with a plus
Difference of cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	Two cubes with a minus
Check	Multiply back out	After every factorization

Naming polynomials

By terms	Example	By degree	Example
Monomial	$7x^3$	Constant (0)	12
Binomial	$2x - 9$	Linear (1)	$3x - 5$
Trinomial	$x^2 - 3x - 4$	Quadratic (2)	$x^2 - 3x - 4$
		Cubic (3), quartic (4)	$2x^3 + x - 1$; $x^4 - 81$

Worked examples

WORKED EXAMPLE 1 · SUBTRACTING POLYNOMIALS

Simplify $(5x^3 - 2x^2 + 4) - (3x^3 + x^2 - 6x + 9)$.

STEP 1 · DISTRIBUTE THE MINUS

$5x^3 - 2x^2 + 4 - 3x^3 - x^2 + 6x - 9$.

STEP 2 · CUBES AND SQUARES

x^3: $5 - 3 = 2$. x^2: $-2 - 1 = -3$.

STEP 3 · X TERMS AND CONSTANTS

x: $0 + 6 = 6$. Constants: $4 - 9 = -5$.

STEP 4 · ANSWER

$2x^3 - 3x^2 + 6x - 5$. The $-6x$ became $+6x$, the step most often missed.

WORKED EXAMPLE 2 · A BINOMIAL TIMES A TRINOMIAL

Multiply $(2x - 1)(x^2 + 4x - 3)$.

STEP 1 · SEND 2X THROUGH

$2x^3 + 8x^2 - 6x$.

STEP 2 · SEND –1 THROUGH

$-x^2 - 4x + 3$.

STEP 3 · COMBINE LIKE TERMS

x^2: $8 - 1 = 7$. x: $-6 - 4 = -10$.

STEP 4 · ANSWER

$2x^3 + 7x^2 - 10x + 3$.

CHECK WITH X = 1

$(1)(1 + 4 - 3) = 2$, and $2 + 7 - 10 + 3 = 2$. They match.

WORKED EXAMPLE 3 · GROUPING

Factor $3x^3 - 12x^2 + 2x - 8$.

STEP 1 · LOOK FOR A GCF OF ALL FOUR

None besides 1, so group in pairs.

STEP 2 · FIRST PAIR

$3x^3 - 12x^2 = 3x^2(x - 4)$.

STEP 3 · SECOND PAIR

$2x - 8 = 2(x - 4)$. The parentheses match.

STEP 4 · PULL OUT THE BINOMIAL

$(x - 4)(3x^2 + 2)$.

CHECK

$(x - 4)(3x^2 + 2) = 3x^3 + 2x - 12x^2 - 8$, the original rearranged.

WORKED EXAMPLE 4 · THE AC METHOD WITH A NEGATIVE C

Factor $8x^2 + 10x - 3$.

STEP 1 · A · C

$8 \cdot (-3) = -24$. Opposite signs; the larger carries the + of b.

STEP 2 · FIND THE PAIR

-1 and 24 (sum 23); -2 and 12 (sum 10). Stop: -2 and 12.

STEP 3 · SPLIT THE MIDDLE

$8x^2 + 12x - 2x - 3$.

STEP 4 · GROUP

$4x(2x + 3) - 1(2x + 3) =$ **$(2x + 3)(4x - 1)$**.

CHECK BY FOIL

$8x^2 - 2x + 12x - 3 = 8x^2 + 10x - 3$.

WORKED EXAMPLE 5 · FACTORING COMPLETELY WITH PATTERNS

Factor $2x^4 - 32$ completely, then factor $64x^3 + 1$.

STEP 1 · GCF FIRST

$2x^4 - 32 = 2(x^4 - 16)$.

STEP 2 · DIFFERENCE OF SQUARES, TWICE

$x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$. So **$2(x^2 + 4)(x + 2)(x - 2)$**; $x^2 + 4$ is a prime sum of squares.

STEP 3 · SUM OF CUBES

$64x^3 = (4x)^3$ and $1 = 1^3$, so a = 4x, b = 1.

STEP 4 · SOAP

$(4x + 1)(16x^2 - 4x + 1)$: same sign, opposite sign, always positive.

TRAPS PEOPLE FALL INTO

- **Adding exponents when adding terms.** $3x^2 + 4x^2 = 7x^2$, not $7x^4$.
- **Flipping only the first sign when subtracting.** The minus reaches every term in the group.
- **Squaring each piece of a binomial.** $(3 + 4)^2 = 49$, but $9 + 16 = 25$. The 2ab term is missing.
- **Skipping the GCF.** It shrinks a · c and often reveals a hidden pattern.
- **Writing nothing when a pair has no common factor.** Write the 1: $3x(2x + 3) + 1(2x + 3)$.
- **Factoring a sum of squares.** $x^2 + 16$ is prime over the reals.
- **Getting the cube signs backward.** SOAP: same, opposite, always positive.
- **Stopping at $3x(x^2 - 9)$.** $x^2 - 9$ still factors into $(x + 3)(x - 3)$.

Chapter 6 self-test · ten items

1. What are the degree and leading coefficient of $7 - 3x^4 + 2x^2$?

A. Degree 4, leading coefficient −3
B. Degree 2, leading coefficient 7
C. Degree 7, leading coefficient −3
D. Degree 4, leading coefficient 3

A. Degree 4, leading coefficient −3 In standard form it is $-3x^4 + 2x^2 + 7$. The sign belongs to the term.

2. Simplify $(2x^2 + 5x - 1) - (x^2 - 3x + 4)$.

A. $3x^2 + 2x - 5$
B. $x^2 + 8x - 5$
C. $x^2 + 2x + 3$
D. $x^2 + 8x + 3$

B. $x^2 + 8x - 5$ Distribute the minus: $-x^2 + 3x - 4$. Then $2 - 1 = 1$, $5 + 3 = 8$, and $-1 - 4 = -5$.

3. Write the product $(x + 6)(2x - 3)$ in standard form.

A. $2x^2 + 15x - 18$

B. $2x^2 - 9x - 18$

C. $2x^2 + 9x - 18$

D. $2x^2 - 18$

C. $2x^2 + 9x - 18$ FOIL: $2x^2 - 3x + 12x - 18$. The middle terms combine to 9x.

4. Write $(3x - 4)^2$ as a trinomial.

A. $9x^2 - 24x - 16$

B. $9x^2 - 12x + 16$

C. $9x^2 + 16$

D. $9x^2 - 24x + 16$

D. $9x^2 - 24x + 16$ $a^2 - 2ab + b^2$ with $a = 3x$, $b = 4$: $9x^2 - 2(3x)(4) + 16$.

5. Factor out the GCF of $15x^5 - 10x^3 + 25x^2$.

A. $x^2(15x^3 - 10x + 25)$

B. $5x(3x^4 - 2x^2 + 5x)$

C. $5x^2(3x^3 - 2x + 5)$

D. $5x^2(3x^3 - 2x^2 + 5)$

C. $5x^2(3x^3 - 2x + 5)$ The GCF of 15, 10 and 25 is 5, and the lowest power of x is x^2.

6. Which factored form equals $x^2 + 2x - 35$?

A. $(x + 5)(x + 7)$

B. $(x + 7)(x - 5)$

C. $(x + 35)(x - 1)$

D. $(x - 7)(x + 5)$

B. $(x + 7)(x - 5)$ Product -35 and sum 2: the pair is 7 and -5.

7. Use the AC method to factor $3x^2 - 11x + 6$.

A. $(x - 3)(3x - 2)$

B. $(x - 2)(3x - 3)$

C. $(x + 3)(3x + 2)$

D. $(x - 6)(3x - 1)$

A. $(x - 3)(3x - 2)$ $a \cdot c = 18$; product 18, sum -11 gives -9 and -2. Then $3x(x - 3) - 2(x - 3)$.

8. Factor $4x^2 - 81$.

A. $(4x + 9)(x - 9)$

B. $(2x + 9)(2x + 9)$

C. $(2x - 9)(2x - 9)$

D. $(2x + 9)(2x - 9)$

D. $(2x + 9)(2x - 9)$ $4x^2 = (2x)^2$ and $81 = 9^2$, with a minus between them.

9. Factor $x^3 - 125$.

A. $(x - 5)(x^2 + 5x + 25)$

B. $(x - 5)(x^2 - 5x + 25)$

C. $(x + 5)(x^2 - 5x + 25)$

D. $(x - 5)(x^2 + 10x + 25)$

A. $(x - 5)(x^2 + 5x + 25)$ Difference of cubes with a = x, b = 5. SOAP gives a minus, then a plus, then a plus.

10. Which polynomial is prime over the real numbers?

A. $x^2 - 25$

B. $x^3 + 8$

C. $x^2 + 25$

D. $x^2 + 10x + 25$

C. $x^2 + 25$ A sum of squares does not factor over the reals. The others are a difference of squares, a sum of cubes and a perfect square.

ONE PAGE

Cram sheet

Everything from chapters 5 and 6 in one place. Cover the right column and see if you can rebuild each row.

Systems at a glance

Item	Rule	Note
Outcomes	One, none, infinitely many	Different slopes; parallel; same line
Names	Consistent independent; inconsistent; consistent dependent	Match the three outcomes
Substitution	Isolate, substitute into the other, solve, back-substitute	Best with a coefficient of ±1
Elimination	Multiply whole equations, add to cancel	Best for Ax + By = C
Endings	0 = nonzero: none; 0 = 0: infinitely many	Same for both methods
3 × 3	Same variable from two pairs, then back-substitute	Check the triple in all three
Inequalities	Solid ≤ ≥; dashed < >; test point; overlap	Flip when dividing by a negative

Formulas

Name	Formula	When to use it
Mixture	$p_1x + p_2y = p_{mix}(x + y)$	Solutions of different strength
Current	$b + c = d/t_{down}$, $b - c = d/t_{up}$	Boats, planes and wind
Square of a sum	$(a + b)^2 = a^2 + 2ab + b^2$	Expanding or factoring
Square of a difference	$(a - b)^2 = a^2 - 2ab + b^2$	Expanding or factoring
Difference of squares	$a^2 - b^2 = (a + b)(a - b)$	Two squares, minus sign
Sum of cubes	$(a + b)(a^2 - ab + b^2)$	SOAP
Difference of cubes	$(a - b)(a^2 + ab + b^2)$	SOAP
AC method	$pq = ac$, $p + q = b$	$ax^2 + bx + c$

Factoring order

Step	What to do
1	Pull out the GCF, including a negative leading coefficient.
2	Count the terms: two (squares or cubes), three (trinomial), four (grouping).
3	Factor each piece again until nothing breaks down.
4	Check by multiplying back out.

Dr. Garg Learning · CC BY-SA 4.0