COLLEGE ALGEBRA · STUDY GUIDE

Exam 2 Study Guide

This guide covers chapters 3 and 4. You will read the coordinate plane, find intercepts and slopes, write lines in three forms and build linear models. Then you will meet functions: the one-output rule, function notation, domain and range, the six toolkit graphs and the moves that shift, stretch and flip them. Work one chapter at a time. Read the "what to know" paragraphs, copy the tables out by hand, then try each worked example with the steps covered. Finish each chapter with its ten self-test items. The cram sheet at the end puts every formula in one place.

WHAT THIS GUIDE COVERS

- Chapter 3 · ordered pairs and quadrants, intercepts, slope and its four kinds, slope-intercept, point-slope and standard form, parallel and perpendicular lines, linear models and average rate of change.
- Chapter 4 · the definition of a function, the vertical line test, function notation, evaluating at numbers and expressions, domain and range, the toolkit functions and transformations.

CHAPTER 3

Graphs and Lines

Read it in full: **Chapter 3 in the textbook**. Practice it in the lab: **Rise Over Run** and **Three Forms of a Line**.

What to know

The coordinate plane is two number lines crossed at zero. The horizontal one is the x-axis, the vertical one is the y-axis, and they meet at the **origin** (0, 0). Every point has an address called an **ordered pair** (x, y): horizontal move first, vertical move second. To plot (4, −3), go 4 right, then 3 down. Saying the moves out loud prevents sign slips.

The axes cut the plane into four **quadrants**, numbered I to IV counterclockwise from the upper right. Signs alone decide the quadrant: (+, +) is I, (−, +) is II, (−, −) is III and (+, −) is IV. A point

with a zero coordinate sits on an axis and is in no quadrant.

An **intercept** is where a graph meets an axis. Every point on the x-axis has $y = 0$, so set $y = 0$ to find the x-intercept. Every point on the y-axis has $x = 0$, so set $x = 0$ to find the y-intercept. Report each as a point, such as (10, 0), not as a lone number. Two intercepts give a fast graph, and a third point is cheap insurance.

Slope measures how fast a line rises or falls as you move right: $m = (y_2 - y_1)/(x_2 - x_1)$, rise over run, valid only when $x_2 \neq x_1$. Either point can be "first," but subtract in the same order top and bottom. Always reduce: 6/4 becomes 3/2, meaning run 2, rise 3.

Every line has one of four slopes. Positive rises left to right. Negative falls. Zero is horizontal, with equation $y = k$. Undefined is vertical, with equation $x = k$, because the run is zero. "Zero slope" and "undefined slope" are different things, so avoid the phrase "no slope." Outside a graph, slope is a rate: miles per hour, dollars per page, always y-units per x-unit.

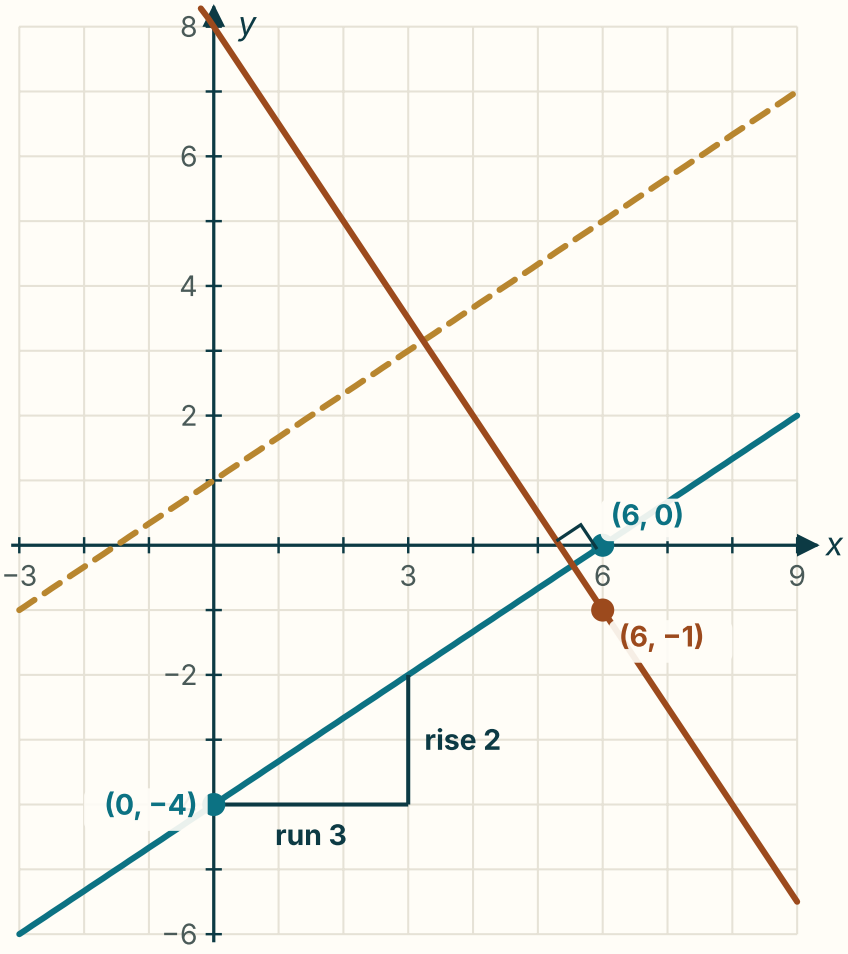
One line can wear three outfits. **Slope-intercept** $y = mx + b$ shows the slope and the y-intercept (0, b). **Point-slope** $y - y_1 = m(x - x_1)$ is the slope formula with the denominator cleared, and it is the form you build with. **Standard form** $Ax + By = C$ uses integers with A non-negative and can hold vertical lines. To read the slope from standard form, solve for y, dividing every term carefully when B is negative.

Parallel lines have equal slopes and different intercepts. Equal slopes and equal intercepts mean the same line twice. **Perpendicular** lines meet at 90°, and their slopes multiply to −1: flip the fraction, then change the sign. A horizontal and a vertical line are perpendicular by geometry, but the product rule cannot be applied to an undefined slope. Put both equations in $y = mx + b$ before comparing.

A **linear model** fits a quantity that changes by the same amount in every equal step. Name the variables with units, find the slope from two data points, write point-slope, simplify, and check with the point you did not use. The slope is the rate and the intercept is the starting value. State the domain, and distrust **extrapolation** far beyond the data. The slope between two data points that are not exactly linear is the **average rate of change**.

LOCK THESE IN

- Ordered pair: horizontal first, vertical second. Points on an axis are in no quadrant.
- x-intercept: set y = 0. y-intercept: set x = 0. Write each as a point.
- $m = (y_2 - y_1)/(x_2 - x_1)$, same order top and bottom.
- Zero slope is horizontal (y = k); undefined slope is vertical (x = k).
- $y = mx + b$, $y - y_1 = m(x - x_1)$ and $Ax + By = C$ describe the same line.
- Parallel: equal slopes. Perpendicular: product −1, flip and negate.
- Check a two-point line with the point you did not use.
- In a model, slope is a rate with units and the intercept is the starting value.



L: 2x − 3y = 12
y = (2/3)x − 4
slope 2/3

Parallel
y = (2/3)x + 1
same slope, new b

Perpendicular
y = −(3/2)x + 8
through (6, −1)

(2/3)(−3/2) = −1

Intercepts of L:
y = 0 gives (6, 0)
x = 0 gives (0, −4)

Flip and negate:
2/3 becomes −3/2

Slope is rise over run, and it decides direction. Parallel lines share a slope with different intercepts. Perpendicular slopes multiply to −1: flip the fraction, then change the sign.

Line formulas and when to use them

Form or rule	Formula	When to use it
Slope	$m = (y_2 - y_1)/(x_2 - x_1)$	Two points are given; $x_2 \neq x_1$
Slope-intercept	$y = mx + b$	Graphing fast; reading slope and intercept; comparing lines
Point-slope	$y - y_1 = m(x - x_1)$	A point and a slope, or two points
Standard	$Ax + By = C$	Tidy integer answers; intercepts; systems
Slope from standard form	$m = -A/B$	$Ax + By = C$ with $B \neq 0$
Horizontal line	$y = k$, slope 0	Every point shares one y-value
Vertical line	$x = k$, slope undefined	Every point shares one x-value
Parallel	$m_2 = m_1$, different b	Building or testing parallel lines
Perpendicular	$m_1 \cdot m_2 = -1$	Flip and negate; not for horizontal with vertical
Average rate of change	$(y_2 - y_1)/(x_2 - x_1)$ over an interval	Data that are not exactly linear

Quadrants and signs

Quadrant	Where	Signs	Example
I	Upper right	(+, +)	(3, 2)
II	Upper left	(−, +)	(−3, 2)
III	Lower left	(−, −)	(−3, −2)
IV	Lower right	(+, −)	(3, −2)
None	On an axis	$x = 0$ or $y = 0$	(0, 5), (−4, 0)

Worked examples

WORKED EXAMPLE 1 · INTERCEPTS FROM STANDARD FORM

Find both intercepts of $3x - 4y = 24$.

STEP 1 · X-INTERCEPT: SET Y = 0

$3x - 4(0) = 24$, so $3x = 24$ and $x = 8$. The point is **(8, 0)**.

STEP 2 · Y-INTERCEPT: SET X = 0

$3(0) - 4y = 24$, so $-4y = 24$ and $y = -6$. The point is **(0, −6)**.

STEP 3 · CHECK BOTH

$3(8) - 4(0) = 24$ and $3(0) - 4(-6) = 24$. Both are true.

GRAPH IT

Plot (8, 0) and (0, −6) and draw the line through them. The slope is $(-6 - 0)/(0 - 8) = 3/4$.

WORKED EXAMPLE 2 · SLOPE FROM TWO POINTS

Find the slope of the line through (−4, 3) and (2, −6).

STEP 1 · LABEL

$x_1 = -4$, $y_1 = 3$, $x_2 = 2$, $y_2 = -6$.

STEP 2 · SUBSTITUTE

$m = (-6 - 3)/(2 - (-4))$.

STEP 3 · TOP AND BOTTOM

Top: −9. Bottom: 2 + 4 = 6.

STEP 4 · DIVIDE AND REDUCE

$m = -9/6 =$ **$-3/2$**. The line falls 3 for every 2 to the right.

CHECK BY SWAPPING

$(3 - (-6))/(-4 - 2) = 9/(-6) = -3/2$. Same slope.

WORKED EXAMPLE 3 · A LINE THROUGH TWO POINTS, IN ALL THREE FORMS

Write the line through (−2, 1) and (3, 11).

STEP 1 · SLOPE

$m = (11 - 1)/(3 - (-2)) = 10/5 = 2$.

STEP 2 · POINT-SLOPE WITH (−2, 1)

$y - 1 = 2(x + 2)$.

STEP 3 · SLOPE-INTERCEPT

$y - 1 = 2x + 4$, so $\mathbf{y = 2x + 5}$.

STEP 4 · STANDARD FORM

Subtract $2x$: $-2x + y = 5$. Multiply by -1 so A is positive: $\mathbf{2x - y = -5}$.

CHECK WITH THE UNUSED POINT

At $x = 3$: $y = 2(3) + 5 = 11$. The line passes through $(3, 11)$.

WORKED EXAMPLE 4 · PARALLEL AND PERPENDICULAR THROUGH A POINT

Line L is $4x + 2y = 6$. Find the lines through $(1, 5)$ that are parallel and perpendicular to L.

STEP 1 · SLOPE OF L

$2y = -4x + 6$, so $y = -2x + 3$ and $m = -2$.

STEP 2 · PARALLEL

Same slope: $y - 5 = -2(x - 1)$, so $\mathbf{y = -2x + 7}$. Its intercept 7 differs from 3, so it is a new line.

STEP 3 · PERPENDICULAR SLOPE

Write -2 as $-2/1$, flip to $-1/2$, negate to $1/2$. Check: $(-2)(1/2) = -1$.

STEP 4 · PERPENDICULAR LINE

$y - 5 = (1/2)(x - 1)$, so $y = (1/2)x - 1/2 + 5 = \mathbf{(1/2)x + 9/2}$.

CHECK THE POINT

$(1/2)(1) + 9/2 = 10/2 = 5$. The line passes through $(1, 5)$.

WORKED EXAMPLE 5 · BUILDING A LINEAR MODEL

A taxi charges a flat fee plus a rate per mile. A 2-mile ride costs \$9.50 and a 10-mile ride costs \$27.50. Write the model and price a 6-mile ride.

STEP 1 · NAME THE VARIABLES

Let d = miles and C = cost in dollars. The data are $(2, 9.50)$ and $(10, 27.50)$.

STEP 2 · SLOPE

$m = (27.50 - 9.50)/(10 - 2) = 18/8 = 2.25$ dollars per mile.

STEP 3 · POINT-SLOPE

$C - 9.50 = 2.25(d - 2)$, so $C = 2.25d - 4.50 + 9.50 = \mathbf{2.25d + 5}$.

STEP 4 · CHECK THE UNUSED POINT

$2.25(10) + 5 = 27.50$. It matches.

STEP 5 · PREDICT AND INTERPRET

2.25(6) + 5 = **$18.50**. Each mile adds $2.25, and $5 is the flat fee at d = 0. The model only makes sense for $d \geq 0$.

TRAPS PEOPLE FALL INTO

- **Writing an intercept as a number.** The x-intercept is the point (8, 0), not "8."
- **Mixing the subtraction order.** $(y_2 - y_1)$ on top needs $(x_2 - x_1)$ below, or the sign flips.
- **Calling a vertical line "zero slope."** Vertical means the run is zero, so the slope is undefined.
- **Flipping without negating.** The perpendicular slope to 2/3 is −3/2, not 3/2.
- **Reading the slope straight off Ax + By = C.** Solve for y first; the slope is −A/B.
- **Forgetting that dividing by a negative changes every term.** −2y = −5x + 8 becomes y = (5/2)x − 4.
- **Calling equal-slope lines parallel when the intercepts also match.** That is one line written twice.
- **Extrapolating far outside the data.** A model describes a range, not a law of nature.

Chapter 3 self-test · ten items

1. A point has a negative x-coordinate and a positive y-coordinate, such as (−4, 9). Where is it?

A. Quadrant IV
B. Quadrant II
C. Quadrant I
D. Quadrant III

B. Quadrant II Negative x and positive y put the point in the upper left, which is Quadrant II.

2. The line 5x − 2y = 30 crosses the x-axis where?

A. (0, 6)

B. (0, −15)

C. (6, 0)

D. (−15, 0)

C. (6, 0) Set y = 0: 5x = 30, so x = 6. The point is (6, 0). (0, −15) is the y-intercept.

3. What is the slope of the line through (−1, 4) and (3, −8)?

A. −3

B. 3

C. 1/3

D. −1/3

A. −3 m = (−8 − 4)/(3 − (−1)) = −12/4 = −3.

4. Two points are (5, −2) and (5, 6). Which statement is correct?

A. Undefined slope; the line is y = 5

B. Zero slope; the line is x = 5

C. Zero slope; the line is y = 5

D. Undefined slope; the line is x = 5

D. Undefined slope; the line is x = 5 The run is 5 − 5 = 0, so the slope is undefined. Both points share x = 5, a vertical line.

5. Which equation is the line with slope −4 through (2, −3)?

A. $y = -4x + 11$

B. $y = -4x + 5$

C. $y = -4x - 5$

D. $y = 4x - 11$

B. $y = -4x + 5$ $y + 3 = -4(x - 2) = -4x + 8$, so $y = -4x + 5$. Check: $-4(2) + 5 = -3$.

6. Rewrite $6x - 3y = 12$ in slope-intercept form. What do you read off?

A. Slope 2, y-intercept (0, −4)

B. Slope 6, y-intercept (0, 12)

C. Slope 2, y-intercept (0, 4)

D. Slope −2, y-intercept (0, 4)

A. Slope 2, y-intercept (0, −4) $-3y = -6x + 12$. Divide every term by −3: $y = 2x - 4$.

7. What slope is perpendicular to $y = (4/7)x - 1$?

A. 4/7

B. −4/7

C. 7/4

D. −7/4

D. −7/4 Flip 4/7 to 7/4, then change the sign. Check: $(4/7)(-7/4) = -1$.

8. How are y = 3x + 2 and 6x − 2y = 8 related?

A. They are the same line
B. They are perpendicular
C. The two lines are parallel
D. They cross, not at 90°

C. The two lines are parallel The second becomes y = 3x − 4. Equal slopes of 3 with different intercepts means parallel.

9. Which is the standard form of the line through (0, 3) and (4, 0)?

A. 4x + 3y = 3
B. 4x + 3y = 12
C. 3x − 4y = 12
D. 3x + 4y = 12

D. 3x + 4y = 12 m = −3/4 and b = 3, so y = −(3/4)x + 3. Multiply by 4: 4y = −3x + 12, then 3x + 4y = 12.

10. A phone is worth V = 640 − 80t dollars after t years. What does the −80 tell you?

A. The value drops $80 per year
B. It is worthless after 80 years
C. The phone started at $80
D. The value rises $80 per year

A. The value drops $80 per year The slope is the rate of change in dollars per year. Negative means the value goes down. It reaches 0 at t = 8.

CHAPTER 4

Introduction to Functions

Read it in full: **Chapter 4 in the textbook**. Practice it in the lab: **The Vertical Line Test** and **Transformations: Shift, Stretch, Reflect**.

What to know

A **relation** pairs inputs with outputs. A **function** is a relation that assigns to each input exactly one output. Repeated outputs are fine: two inputs may share an output. What breaks a function is one input with two different outputs. The input is the **independent variable**, usually x, and the output is the **dependent variable**, usually y.

The test is the same in every form. In ordered pairs or a table, look for a repeated input with different outputs. In a mapping, exactly one arrow must leave each input. In an equation, plug in an x and see whether one y comes out: $y^2 = x$ fails, because $x = 9$ gives $y = 3$ and $y = -3$. On a graph, use the **vertical line test**: if any vertical line crosses the graph twice, it is not a function. The test is the definition, drawn.

Function notation f(x) is read "f of x." It is never f times x. f(3) means run 3 through the rule. To evaluate, replace every x with the input in parentheses and simplify by order of operations. f(3) = 8 says the point (3, 8) is on the graph. For an expression input such as f(a + 1), substitute the whole quantity, and remember $(a + 1)^2 = a^2 + 2a + 1$.

f(0) and f(x) = 0 are opposite questions. f(0) gives an input and asks for the output: the y-intercept. f(x) = 0 gives the output and asks for the input: the x-intercepts. Going backwards, from an output to an input, means solving an equation.

The **domain** is every allowed input. The **range** is every output produced. In this course only two things shrink a domain: a denominator that would be zero, and a negative under an even root. Find the domain by exclusion: assume all reals, then throw those values out. A zero numerator is fine. When a root sits in a denominator, both rules apply, so $\sqrt{(x - 1)}$ in a denominator needs $x > 1$.

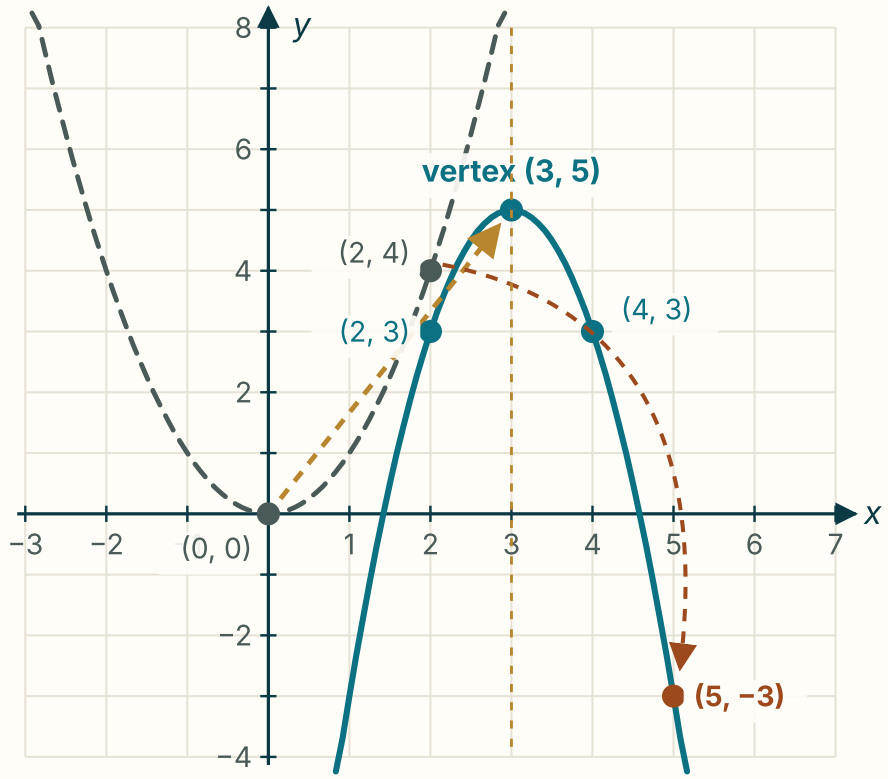
Write domains and ranges in interval notation. From a graph, read the domain left to right and the range bottom to top. For $y = x^2 - 4$ the domain is $(-\infty, \infty)$ and the range is $[-4, \infty)$, with a bracket because -4 is reached at the vertex.

Six **toolkit functions** show up again and again: x, x^2, |x|, $\sqrt{x}$, x^3 and 1/x. x^2 and |x| fold over the y-axis and have range $[0, \infty)$. x and x^3 reach every real output. $\sqrt{x}$ has domain and range $[0, \infty)$. 1/x

leaves out 0 from both its domain and range, so it has two branches and two asymptotes, $x = 0$ and $y = 0$.

Transformations move a toolkit graph. Changes outside the function are vertical and do what they say: $f(x) + k$ moves up k, $a \cdot f(x)$ stretches by a, and $-f(x)$ flips over the x-axis. Changes inside, next to the x, are horizontal and reversed: $f(x - h)$ moves right h, and $f(-x)$ flips over the y-axis. To track a point, solve the inside for the new x, then apply the outside changes to the old y. Check with two or three evaluated points.

> **LOCK THESE IN**
>
> - One input, exactly one output. Repeated outputs are allowed.
> - Two hits on one vertical line means not a function.
> - $f(x)$ is "f of x," never multiplication; substitute in parentheses.
> - $f(0)$ is the y-intercept; $f(x) = 0$ gives the x-intercepts.
> - Domain by exclusion: no zero denominators, no negatives under even roots.
> - Toolkit: x, x^2, $|x|$, $\sqrt{x}$, x^3, $1/x$.
> - Outside: vertical and literal. Inside: horizontal and reversed.
> - Track a point: solve inside for x, apply outside changes to y.



- - - $f(x) = x^2$

——— $g(x) = -2(x - 3)^2 + 5$

1. x − 3 inside: right 3
2. 2 outside: stretch by 2
3. minus: flip over x-axis
4. +5 outside: up 5

Track (2, 4):
$x - 3 = 2$, so $x = 5$
$4(-2) + 5 = -3$
lands at (5, −3)

Range $(-\infty, 5]$

Inside changes are horizontal and reversed; outside changes are vertical and literal. The vertex moves from (0, 0) to (3, 5), and the point (2, 4) lands at (5, −3). Checking $g(5) = -3$ confirms it.

Transformations and when they apply

Form	Effect on the graph	When you see it
$f(x) + k$	Up k (down if $k < 0$)	A number added outside
$f(x - h)$	Right h (left if written $x + h$)	A number added to x inside
$-f(x)$	Reflect across the x-axis	A minus sign outside
$f(-x)$	Reflect across the y-axis	A minus sign on x inside
$a \cdot f(x)$, $a > 1$	Vertical stretch, narrower	A multiplier outside
$a \cdot f(x)$, $0 < a < 1$	Vertical compression, wider	A fraction multiplier outside
$f(bx)$, $b > 1$	Horizontal compression by 1/b	A multiplier on x inside
Point tracking	(x_0, y_0) on f goes to $(x_0 + h, a \cdot y_0 + k)$ on $a \cdot f(x - h) + k$	Finding where one point lands

The six toolkit functions

Function	Domain	Range	Shape
$f(x) = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	Line through the origin
$f(x) = x^2$	$(-\infty, \infty)$	$[0, \infty)$	Parabola, vertex (0, 0)
$f(x) = \lvert x \rvert$	$(-\infty, \infty)$	$[0, \infty)$	V with corner at (0, 0)
$f(x) = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$	Half parabola on its side
$f(x) = x^3$	$(-\infty, \infty)$	$(-\infty, \infty)$	S-curve through the origin
$f(x) = 1/x$	$x \neq 0$	$y \neq 0$	Two branches, asymptotes $x = 0$ and $y = 0$

Worked examples

WORKED EXAMPLE 1 · IS IT A FUNCTION?

Decide whether $\{(-1, 2), (0, 2), (3, 5), (-1, 4)\}$ is a function, and whether $x^2 + y^2 = 25$ makes y a function of x.

STEP 1 · LIST THE INPUTS OF THE SET

$-1, 0, 3, -1$. The input -1 repeats.

STEP 2 · COMPARE ITS OUTPUTS

$(-1, 2)$ and $(-1, 4)$: one input, two outputs. **Not a function.** The shared output 2 for -1 and 0 was never the problem.

STEP 3 · TEST THE EQUATION WITH ONE INPUT

Let $x = 3$: $9 + y^2 = 25$, so $y^2 = 16$ and $y = 4$ or $y = -4$.

STEP 4 · CONCLUDE

One input gave two outputs, so the circle is **not a function**. The vertical line $x = 3$ hits it at $(3, 4)$ and $(3, -4)$.

WORKED EXAMPLE 2 · EVALUATING AT A NUMBER AND AN EXPRESSION

Let $f(x) = 2x^2 - 3x + 4$. Find $f(-3)$ and $f(a - 1)$.

STEP 1 · SUBSTITUTE –3 IN PARENTHESES

$f(-3) = 2(-3)^2 - 3(-3) + 4$.

STEP 2 · SIMPLIFY

$2(9) + 9 + 4 = 18 + 9 + 4 =$ **31**. The point $(-3, 31)$ is on the graph.

STEP 3 · SUBSTITUTE THE WHOLE EXPRESSION

$f(a - 1) = 2(a - 1)^2 - 3(a - 1) + 4$.

STEP 4 · EXPAND

$2(a^2 - 2a + 1) - 3a + 3 + 4 = 2a^2 - 4a + 2 - 3a + 7 =$ **$2a^2 - 7a + 9$**.

CHECK WITH A = 2

Our answer: $8 - 14 + 9 = 3$. Directly: $f(1) = 2 - 3 + 4 = 3$. They match.

WORKED EXAMPLE 3 · TWO DOMAINS

Find the domain of $f(x) = (x - 1)/(x^2 - 16)$, then the domain and range of $g(x) = \sqrt{(12 - 3x)}$.

STEP 1 · DENOMINATOR HAZARD

$x^2 - 16 = 0$ gives $x^2 = 16$, so $x = 4$ or $x = -4$.

STEP 2 · DOMAIN OF F

$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$. The numerator being zero at $x = 1$ is fine.

STEP 3 · ROOT HAZARD

$12 - 3x \geq 0$, so $-3x \geq -12$. Divide by -3 and flip: $x \leq 4$.

STEP 4 · DOMAIN AND RANGE OF G

Domain **$(-\infty, 4]$**. The square root never gives a negative, and $g(4) = 0$, so the range is **$[0, \infty)$**.

TEST POINTS

$g(0) = \sqrt{12}$, a real number. $g(5) = \sqrt{(-3)}$, not real, correctly excluded.

WORKED EXAMPLE 4 · DECODING A TRANSFORMATION AND TRACKING A POINT

Describe $h(x) = 2|x - 1| - 3$ as a change of $|x|$, and find where $(-1, 1)$ lands.

STEP 1 · INSIDE

$x - 1$ inside the bars: shift **right 1**.

STEP 2 · OUTSIDE

The 2 stretches vertically by 2, and the -3 shifts **down 3**. The corner moves from $(0, 0)$ to $(1, -3)$.

STEP 3 · TRACK THE X

Solve $x - 1 = -1$, so $x = 0$.

STEP 4 · TRACK THE Y

Old y is 1. Times 2 is 2, minus 3 is −1. The point lands at **(0, −1)**.

CHECK

h(0) = 2|−1| − 3 = 2 − 3 = −1. The range is [−3, ∞).

TRAPS PEOPLE FALL INTO

- **Rejecting repeated outputs.** {(1, 4), (2, 4)} is a function. Only a repeated input with different outputs fails.
- **Reading f(x) as f times x.** The parentheses hold the input.
- **Substituting without parentheses.** f(−2) for x^2 is $(-2)^2 = 4$, not −4.
- **Writing $(a + 1)^2 = a^2 + 1$.** It is $a^2 + 2a + 1$.
- **Excluding a value that zeros the numerator.** Only a zero denominator is excluded.
- **Moving f(x − 3) left.** Inside changes reverse: x − 3 moves the graph right 3.
- **Changing x when the change is outside.** −3√x keeps every x and multiplies every y by −3.
- **Putting a bracket on an endpoint that is never reached.** Use a bracket only when the value is actually attained.

Chapter 4 self-test · ten items

1. Exactly one of these sets of ordered pairs breaks the function rule. Which one?

A. {(1, 2), (2, 2), (3, 2)}

B. {(0, 0), (1, 1), (2, 4)}

C. {(4, 1), (5, 2), (4, 3)}

D. {(−1, 5), (1, 5), (2, 6)}

C. {(4, 1), (5, 2), (4, 3)} The input 4 is paired with both 1 and 3. Repeated outputs, as in the other sets, are allowed.

2. Which graph fails the vertical line test?

A. The upward parabola $y = x^2$
B. A circle centered at (0, 0)
C. The slanted line $y = -3x + 1$
D. The V-shaped graph $y = |x|$

B. A circle centered at (0, 0) A vertical line through a circle hits it twice, giving one x two y-values.

3. Let $f(x) = x^2 - 5x$. What is $f(-2)$?

A. 6
B. −14
C. −6
D. 14

D. 14 $(-2)^2 - 5(-2) = 4 + 10 = 14$. Writing $-2^2 = -4$ gives the wrong answer 6.

4. Let $g(x) = 4x - 9$. For which x is $g(x) = 15$?

A. $x = 6$
B. $x = 51$
C. $x = -6$
D. $x = 1.5$

A. x = 6 Solve $4x - 9 = 15$: $4x = 24$, $x = 6$. The value 51 is g(15), the opposite question.

5. Let f(x) = 3x + 2. What is f(h + 1)?

A. h + 3

B. 3h + 5

C. 3h + 3

D. 3h + 1

B. 3h + 5 3(h + 1) + 2 = 3h + 3 + 2 = 3h + 5.

6. Which inputs may f(x) = 5/(x + 6) accept?

A. (−∞, −6) ∪ (−6, ∞)

B. (−6, ∞), right of −6

C. [−6, ∞), with −6 kept

D. (−∞, 6) ∪ (6, ∞)

A. (−∞, −6) ∪ (−6, ∞) The denominator is zero at x = −6, so only −6 is excluded.

7. For g(x) = √(2x + 10), which interval lists every allowed input?

A. (−∞, −5]

B. (−5, ∞)

C. [−5, ∞)

D. [5, ∞)

C. [−5, ∞) 2x + 10 ≥ 0 gives x ≥ −5. The endpoint is allowed because √0 = 0.

8. Which outputs does $f(x) = -x^2 + 6$ produce?

A. $[-6, \infty)$

B. $[6, \infty)$

C. $(-\infty, \infty)$

D. $(-\infty, 6]$

D. $(-\infty, 6]$ $-x^2$ is never positive, so the largest output is 6 at $x = 0$ and the graph opens downward.

9. Where is the vertex of $y = (x - 4)^2 + 1$?

A. (1, 4)

B. (−4, 1)

C. (4, −1)

D. (4, 1)

D. (4, 1) Inside $x - 4$ moves the graph right 4. Outside +1 moves it up 1.

10. The point (9, 3) is on $y = \sqrt{x}$. Where does it land on $g(x) = \sqrt{(x + 2)} - 5$?

A. (7, −2)

B. (7, 8)

C. (11, −2)

D. (11, 8)

A. (7, −2) Inside: $x + 2 = 9$, so $x = 7$. Outside: $3 - 5 = -2$. Check: $\sqrt{9} - 5 = -2$.

ONE PAGE

Cram sheet

Everything from chapters 3 and 4 in one place. Cover the right column and see if you can rebuild each row.

Formulas

Name	Formula	When to use it
Slope	$m = (y_2 - y_1)/(x_2 - x_1)$	Two points, $x_2 \neq x_1$
Slope-intercept	$y = mx + b$	Graphing; reading m and b
Point-slope	$y - y_1 = m(x - x_1)$	Building a line
Standard	$Ax + By = C$	Integer form; slope $-A/B$
Perpendicular	$m_2 = -1/m_1$	$m_1 \neq 0$
Linear model	$y = (\text{rate})x + (\text{start})$	Constant change per step
Transformation	$a \cdot f(x - h) + k$	Right h, stretch a, up k
Point tracking	(x_0, y_0) to $(x_0 + h, a \cdot y_0 + k)$	Where one point lands

Rules to know

Topic	Rule	Note
Intercepts	Set y = 0 for x-intercept; x = 0 for y-intercept	Report as points
Four slopes	Positive, negative, zero (y = k), undefined (x = k)	Zero is not undefined
Parallel	Equal slopes, different intercepts	Same b means same line
Function	Each input, exactly one output	Vertical line test
Notation	f(3) = 8 means (3, 8) is on the graph	f(0) vs f(x) = 0
Domain	Exclude zero denominators and negatives under even roots	Root in a denominator: strict inequality
Range from a graph	Read bottom to top	Bracket only when attained
Inside vs outside	Inside: horizontal, reversed. Outside: vertical, literal	f(−x) flips left-right

Toolkit at a glance

Function	Domain	Range
x and x^3	$(-\infty, \infty)$	$(-\infty, \infty)$
x^2 and $\lvert x \rvert$	$(-\infty, \infty)$	$[0, \infty)$
$\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
1/x	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$

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